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ALGORITHMIC BARGAINING: A DYNAMIC PRICING MODEL FOR PRODUCTS AND SERVICES ACROSS ONLINE PLATFORMS

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ABSTRACT

This article presents a comprehensive study of algorithmic bargaining within dynamic pricing systems, focusing on its usability, challenges, and broader market implications. Algorithmic bargaining, increasingly used in industries such as e-commerce, retail, and transportation, allows companies to dynamically adjust prices based on real-time data, consumer behavior, and market conditions. However, its widespread adoption raises significant questions regarding fairness, transparency, and regulatory oversight. Drawing on studies from fields like game theory, behavioral economics, and data science, this research explores the impacts of algorithmic bargaining on both businesses and consumers. While machine learning and deep reinforcement learning technologies enhance pricing efficiency, they also present risks of consumer harm and market manipulation. This paper critically examines the ethical implications, regulatory responses, and potential consequences for competition and consumer welfare. Through an analysis of existing literature, including case studies from various industries, this study provides a balanced evaluation of the benefits and drawbacks of algorithmic bargaining, encouraging a deeper understanding of its multifaceted role in modern pricing strategies.

1. Introduction

The rapid advancement of technology has significantly transformed pricing strategies in e-commerce and other digital platforms, with algorithmic pricing systems becoming the norm. These systems leverage data-driven algorithms to dynamically adjust prices in real-time based on factors like consumer behavior, demand fluctuations, competitor pricing, and inventory levels - core components of behavioral economics [1]. While dynamic pricing has proven effective for optimizing profits and improving market efficiency, it has also raised ethical concerns regarding fairness, transparency, and consumer trust.

Dynamic pricing has been widely adopted in industries such as retail, travel, hospitality, and entertainment [2], with companies like Amazon and Uber using these systems to tailor prices based on demand predictions. However, the unpredictable and opaque nature of real-time price changes has led to skepticism among consumers, sparking discussions about the potential biases these systems may introduce, especially in regard to disadvantaging certain consumer groups [3].

Despite the efficiency of dynamic pricing, these systems typically operate in a one-sided manner, where prices are set by the seller's algorithm with little to no consumer input [4]. This has given rise to the concept of algorithmic bargaining, an emerging paradigm that allows consumers to actively participate in price negotiations within parameters set by the seller's algorithm. Rather than passively accepting price fluctuations, consumers are empowered to engage with pricing systems, fostering a more personalized and interactive shopping experience. Algorithmic bargaining represents a shift toward a more collaborative approach to price setting, offering a potential solution to some of the limitations of current dynamic pricing models.

However, the implementation of algorithmic bargaining raises important questions about usability, fairness, and consumer satisfaction, which remain largely unexplored. There are also significant technical and economic challenges to consider, such as balancing flexibility with profitability and maintaining consumer trust in the bargaining process.

This research aims to address these gaps by exploring the usability and market implications of algorithmic bargaining within dynamic pricing systems. By reviewing existing literature and analyzing the challenges of integrating bargaining capabilities into digital pricing systems, this study seeks to provide insights into whether algorithmic bargaining can enhance transparency, consumer engagement, and fairness in the digital marketplace.

2. Evolution of Dynamic Pricing Systems

In the early days, dynamic pricing was a relatively simple concept. Merchants would adjust their prices based on factors such as the time of day, the customer's perceived bargaining power, and the availability of competing products. This informal approach laid the groundwork for the more structured and data-driven systems that emerged later.

The advent of the digital age marked a significant turning point in the evolution of dynamic pricing. Airlines [4] and hotels were among the first industries to adopt dynamic pricing strategies, leveraging computer systems to analyze demand patterns and adjust prices accordingly. This early adoption demonstrated the potential of dynamic pricing to optimize revenue and improve operational efficiency.

As technology continued to advance, dynamic pricing systems became increasingly sophisticated. The development of advanced algorithms and data mining techniques enabled businesses to make more accurate and timely price adjustments. These innovations allowed for a deeper understanding of customer behavior, market trends, and competitive dynamics.

In recent years, the integration of artificial intelligence (AI) and machine learning has further revolutionized dynamic pricing. AI-powered algorithms can analyze vast amounts of data to identify patterns and trends that may be difficult for humans to detect. Additionally, machine learning models can continuously learn and adapt to changing market conditions, improving the accuracy and effectiveness of pricing decisions.

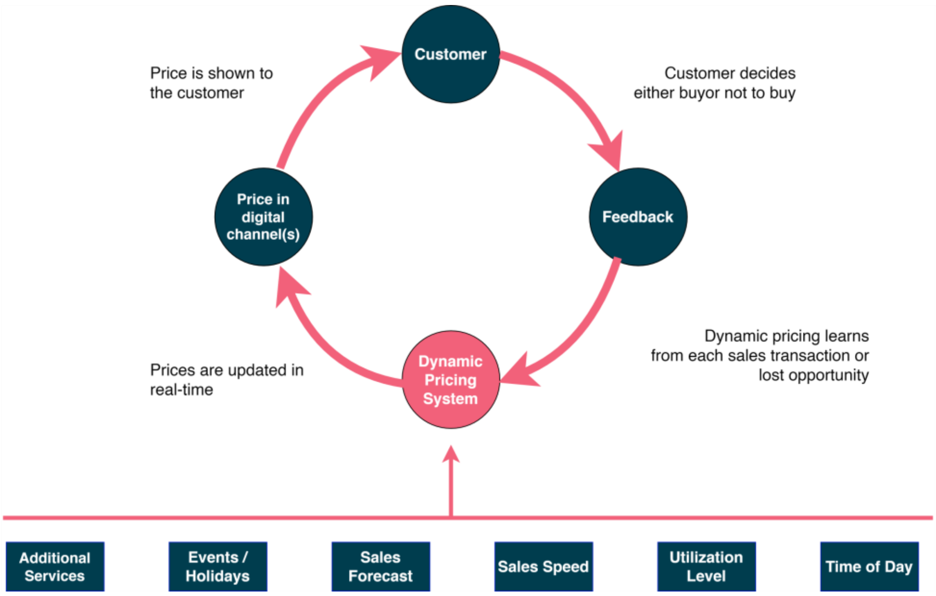

Fig. 1 The online reinforcement learning based dynamic pricing loop [5]

Despite its benefits, dynamic pricing has faced several challenges. One major concern is the potential for price discrimination, where customers are charged different prices based on their perceived ability to pay. This practice can lead to negative consumer sentiment and may attract regulatory scrutiny. In addition, the complexity of implementing dynamic pricing systems can be a barrier for businesses, as they require substantial investments in technology and data expertise. Moreover, the quality and availability of data are critical - flawed or incomplete data can lead to suboptimal pricing decisions, resulting in either lost revenue or dissatisfied customers. Another challenge is that dynamic pricing systems often fail to fully account for external factors like macroeconomic conditions or unforeseen competitive pressures, which can dramatically alter demand.

As dynamic pricing continues to evolve, the integration of advanced technologies like the Internet of Things (IoT) and blockchain is poised to usher in a new era of algorithmic bargaining. This paradigm shift enables real-time, decentralized pricing adjustments that consider a wider array of factors such as inventory levels, customer preferences [2], and social media sentiment. By allowing consumers to actively negotiate prices [6], algorithmic bargaining mirrors traditional haggling practices, enhancing consumer engagement and addressing concerns related to price discrimination. This innovative approach not only empowers buyers but also fosters a more transparent and interactive marketplace, ultimately redefining the dynamics of pricing strategies in the digital commerce landscape.

3. Algorithmic Bargaining: A New Paradigm

Algorithmic bargaining introduces a novel approach to pricing, blending the flexibility of traditional haggling with modern AI-driven algorithms. While dynamic pricing typically uses automated systems to adjust prices based on predefined data points such as supply, demand, and historical trends, algorithmic bargaining adds a real-time, interactive component. This shift enables customers to engage directly in price negotiations, similar to the informal bargaining that is common in traditional marketplaces, particularly in Asia and other regions where haggling is embedded in the culture. However, this model is not without its complexities and potential drawbacks, both from a consumer and business perspective.

One of the significant advantages of algorithmic bargaining is its ability to enhance customer engagement. By allowing buyers to negotiate prices, the system can offer a more personalized and potentially more satisfying experience. This participatory aspect mirrors the experience of in-person bargaining, where both the buyer and seller work to reach a mutually agreeable price. AI algorithms are able to analyze consumer data, such as purchase history, browsing patterns, and even real-time behavior offering counteroffers within defined thresholds, much like a human negotiator. This interactivity can lead to better customer satisfaction and may foster a sense of empowerment, as consumers feel they have more control over their purchasing decisions [6].

However, not all consumers may embrace this new model. For many, the fixed pricing of traditional e-commerce provides simplicity and clarity, eliminating the perceived hassle of negotiations. Algorithmic bargaining may be seen as time-consuming or even manipulative by some, especially in cultures where fixed pricing is the norm and haggling is viewed as uncomfortable or undesirable [7]. Moreover, customers might fear that businesses are using algorithms to artificially inflate the initial prices before offering "discounts" through negotiation, which could generate mistrust. Balancing consumer trust with interactive pricing systems will be critical for the widespread acceptance of algorithmic bargaining.

From a business perspective, the implementation of algorithmic bargaining systems is far more complex than traditional dynamic pricing. It requires not only the development of sophisticated AI models capable of real-time negotiation but also the integration of vast amounts of data, including customer behavior, market conditions, and inventory levels. The practicality of this system may also be questioned. Will customers negotiate every transaction? Or will algorithmic bargaining be limited to certain industries or high-ticket items where negotiation is traditionally more accepted, such as automobiles or luxury goods? For businesses, the added complexity may require significant investments in technology and expertise, which could limit its feasibility for smaller companies or those with tight profit margins [2].

Furthermore, algorithmic bargaining raises important questions about its broader applicability. While it may work well in industries where negotiation is already a norm, its adoption in industries where fixed pricing is expected may meet resistance. How businesses implement this model? Whether as an optional feature or the default system? Will play a crucial role in its success? As a result, algorithmic bargaining may not completely replace traditional pricing models but could instead complement them, giving businesses more flexibility to adapt to diverse customer preferences.

The success of algorithmic bargaining will also depend on its ability to address some of the inherent challenges faced by dynamic pricing systems. For instance, external factors such as macroeconomic conditions, competitive pressures, or sudden market shifts may still impact pricing models in unpredictable ways. AI models that power algorithmic bargaining must be able to adjust quickly to these conditions, ensuring that businesses can remain competitive while still providing fair and engaging pricing structures to their customers.

In conclusion, while algorithmic bargaining represents a new frontier in dynamic pricing, it remains an experimental concept. Its success will depend not only on technological advancements but also on consumer acceptance and practical implementation strategies. As this paradigm evolves, businesses must carefully consider how to integrate it into their pricing models, balancing the benefits of increased customer engagement with the potential risks of complexity and consumer skepticism. Algorithmic bargaining could redefine the future of e-commerce by merging the best of human negotiation and AI-driven efficiency, but its practical feasibility remains to be fully explored.

Transitioning from the conceptual framework of algorithmic bargaining, it is essential to understand the technological underpinnings that enable this shift. The following sections delves into the AI technologies, machine learning algorithms and real-time data analytics that serve as the foundation for this innovative approach to pricing.

4. Technological Foundations of Algorithmic Bargaining

Algorithmic bargaining leverages AI and machine learning to create a dynamic and interactive pricing experience. This section details the underlying technological mechanisms that enable real-time negotiations between the system and consumers.

The process begins with data collection. Utilizing various tracking mechanisms, including cookies, the algorithm gathers comprehensive user information, such as browsing history, purchase patterns, and other relevant data points. This data is crucial for understanding individual consumer preferences and estimating their willingness to pay. While the term "purchasing power parity" (PPP) is sometimes used colloquially, it's more accurate to describe this as estimating a customer's specific valuation of a product or service based on their revealed preferences.

Based on this gathered information, along with current market conditions (e.g., competitor pricing, demand fluctuations), the algorithm establishes an initial price. This leverages similar dynamic pricing principles discussed earlier, but incorporates individualized insights derived from user data. Crucially, a predetermined minimum price, known as the cost price (CP), is established to safeguard against losses for the seller. In addition, a marked price (MP), typically representing the standard or list price, can also be defined.

The core innovation of algorithmic bargaining lies in its interactive nature. This "interactive bargaining" component allows consumers to submit bids in real-time, engaging in a dynamic negotiation with the system's AI agent.

The AI agent, trained using machine learning, particularly reinforcement learning techniques, evaluates user bids against predefined business targets and profit margins. Initially, the algorithm is likely to decline most customer offers to maintain profitability. However, the system is designed to strategically accept certain bids, even those below the CP in specific instances. This "reward" scenario, where a customer receives a price significantly below market value, creates a positive reinforcement loop, encouraging repeat engagement and promoting organic word-of-mouth advertising. This principle is supported by research in behavioral economics and studies on reward-driven behavior, such as the "Monkey Paper" [8], which demonstrates the powerful influence of rewards on decision-making, even in the face of potentially suboptimal outcomes. By offering occasional deep discounts, businesses can tap into this reward mechanism, fostering customer loyalty and driving traffic to their platform.

The system's decision-making process utilizes a probabilistic approach with adjustable parameters to determine the offered price. The algorithm doesn't choose prices entirely randomly but uses predefined frequencies to manage the distribution of offers. For example, a business might configure the following frequency distribution where CP is cost price of product and MP is Marked Price of product:

$Price > MP$: 5% (Offer prices above the marked price rarely to maintain perceived value)

$CP < Price < MP$: 90% (Rarely prices above the MP rarely to maintain perceived value)

$Price < CP$: 5% (Occasionally accepts price below CP as a "reward" to stimulate engagement)

These percentages are customizable, allowing businesses to control the frequency and magnitude of rewards. This asynchronous, probabilistic approach, combined with controlled price frequencies, adds an element of chance and strategy to the bargaining process, enhancing the user experience by creating the perception of "winning" a deal. When a user's bid aligns with the criteria for a reward price, the system congratulates them, triggering a dopamine release that reinforces positive associations with the platform and encourages future interactions.

While complex, this system strives to balance profitability with user engagement, creating a dynamic and personalized shopping experience. Further research and real-world implementation are needed to fully understand the long-term implications and efficacy of this novel pricing paradigm.

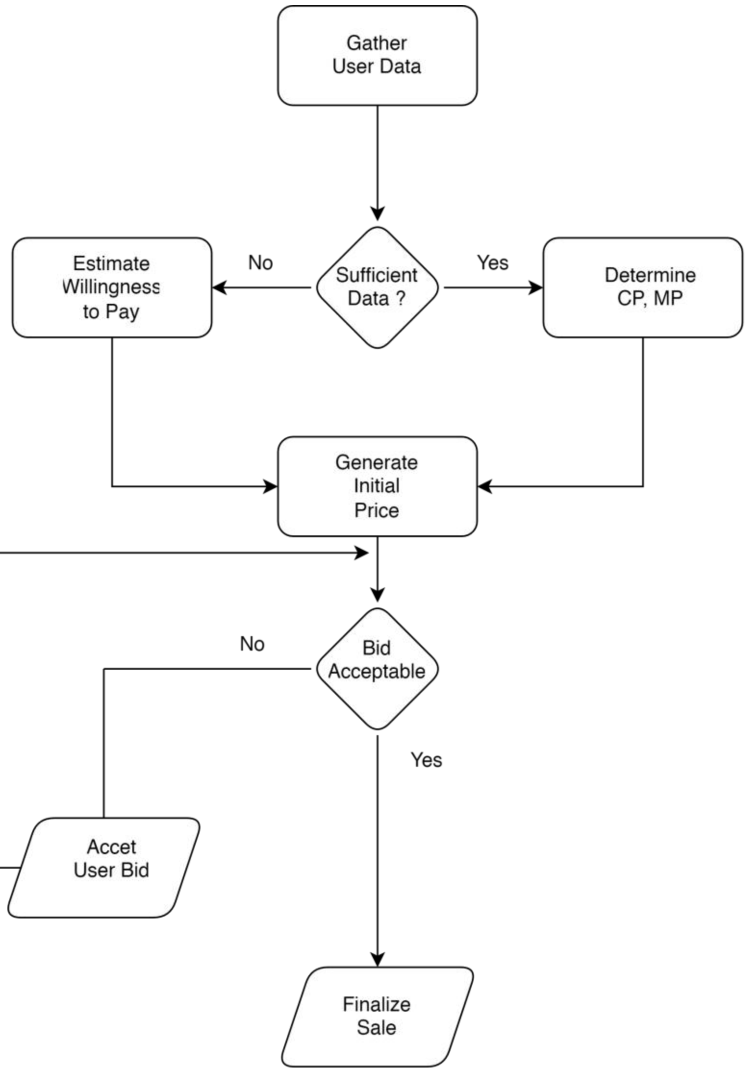

Fig. 2 Flowchart depicting the Algorithmic Bargaining Process. The flowchart should visualize the steps described above, from data collection to price determination, user interaction, and the reward mechanism, incorporating the MP and price frequency logic.

```
import random

def bargain(bid, cost, marked):
"""Algorithmic bargaining with probabilistic pricing."""
frequencies = { # Chances for each price range
"above_marked": 0.05, # 5% chance of price > marked price
"within_range": 0.90, # 90% chance of cost < price < marked
"below_cost": 0.05 # 5% chance of price < cost (reward)
}

rand_num = random.random()
if rand_num < frequencies["above_marked"]:
price = marked * (1 + random.uniform(0.05, 0.2)) # Add 5-20%
elif rand_num < frequencies["above_marked"] + frequencies["within_range"]:
price = random.uniform(cost, marked)
else:
price = cost * (1 - random.uniform(0.05, 0.15)) # Subtract 5-15%

accepted = bid >= price
return price, accepted
```

Fig. 3 Python code demonstrating algorithmic bargaining. The function bargain() simulates a negotiation, taking the user's bid (bid), the seller's minimum price (cost), and the marked price (marked) as input. It returns the algorithmically determined price (price) and whether the bid was accepted (accepted). Price frequencies are controlled by the frequencies dictionary, allowing occasional reward pricing below cost.

5. Bargaining in E-Commerce: Usability and Implementation

The integration of bargaining mechanisms into e-commerce platforms marks a notable evolution from traditional fixed-price models, merging dynamic pricing with heightened consumer interaction. While the potential for enhanced user engagement and revenue optimization is clear, implementing such systems entails addressing significant technical, operational, and ethical challenges.

Bargaining systems rely heavily on their usability [9], which is rooted in offering interactive and customized experiences for consumers. Unlike static pricing approaches, these mechanisms adapt dynamically to user behavior, preferences, and inferred purchasing power. This adaptability begins with robust data collection and analysis. By leveraging digital footprints such as browsing histories, prior purchase records, demographic information, and inferred behavioral patterns, the system tailors price negotiations to individual users. This personalization fosters inclusivity and encourages users to perceive the process as equitable and responsive to their needs.

At the heart of implementation lies the bargaining algorithm, an intricate component that integrates data analytics, pricing strategies, and real-time decision-making processes. These algorithms gather initial inputs through tools such as cookies, user profiles, and session tracking. Based on this data, they generate initial price ranges and continuously adjust negotiation strategies. Advanced pricing models, often employing machine learning techniques [10] like regression analysis and reinforcement learning, refine the system's responses to optimize profitability while maintaining user satisfaction.

Predictive analytics plays a pivotal role in enabling the system to anticipate user behavior and adapt to market dynamics. By analyzing factors such as competitor pricing, seasonal trends, and inventory levels, the algorithm dynamically adjusts its negotiation parameters to stay competitive and relevant [11]. However, ensuring such adjustments align with the platform's overarching goals and customer expectations requires constant fine-tuning and monitoring.

The user interface is a cornerstone of effective bargaining systems, as it mediates the interaction between users and the underlying algorithms. To foster engagement, interfaces may include conversational negotiation tools, chatbots, or gamified pricing models. For example, a chatbot interface that simulates human-like negotiation behavior can create an immersive experience. However, transparency and clarity are essential to avoid user confusion or mistrust. Interfaces must clearly outline negotiation rules, such as minimum acceptable prices and maximum discounts, to maintain a fair and informed interaction. Providing users with insights into how prices are determined can further enhance trust and satisfaction.

Backend integration is another critical aspect of implementation. E-commerce platforms must ensure that their bargaining systems seamlessly connect with payment gateways, inventory management systems, and customer support tools. For example, when a customer successfully negotiates a price, the system should immediately update backend records and reflect the changes in the user's checkout process. Such synchronization minimizes friction and ensures a seamless shopping experience.

Despite their potential, bargaining systems face several challenges that necessitate careful consideration. Scalability is a primary concern, as platforms must manage high volumes of concurrent negotiations without compromising performance or user experience. Additionally, fairness is an essential consideration. Algorithms must avoid discriminatory pricing practices that could alienate or disadvantage specific user groups [12]. To address these concerns, regular audits and algorithmic adjustments are necessary to ensure equitable treatment across a diverse user base.

Privacy concerns also loom large in the implementation of bargaining systems. As these systems depend on extensive data collection and analysis, adherence to data protection regulations such as the GDPR is imperative. Platforms must clearly communicate data usage policies to users and provide mechanisms for data control and opt-outs.

The integration of bargaining mechanisms into e-commerce platforms represents a significant advancement, blending dynamic pricing with interactive and personalized customer experiences. While the potential benefits are vast, their realization depends on strategic implementation, robust system design, and a commitment to addressing technical, ethical, and regulatory challenges. These considerations underscore the need for a balanced approach that prioritizes both operational efficiency and user trust.

6. Consumer Experience and Behavioral Responses

In the evolving landscape of online commerce, algorithmic bargaining introduces a novel dimension to consumer interaction, setting it apart from both static and dynamic pricing systems. Consumer responses to this approach are likely to be complex and influenced by a variety of factors, contrasting significantly with reactions to traditional pricing models. The interactive nature of bargaining creates an opportunity for personalized engagement that goes beyond the predictability of static prices or the opacity of dynamic pricing.

A key factor influencing consumer experience is the perceived fairness of the bargaining process. Consumers are highly sensitive to the transparency and perceived legitimacy of pricing mechanisms [13]. The effectiveness of algorithmic bargaining is substantially mediated by its ability to communicate computational logic clearly, fostering trust through simplified explanations of pricing decisions. When platforms successfully establish this transparency, they mitigate potential negative perceptions that might arise from seemingly arbitrary price adjustments.

The psychological dimensions of algorithmic bargaining reveal intricate motivational responses. By involving consumers directly in the pricing process, these systems transform purchasing from a unilateral transaction to a collaborative engagement. This approach appeals to consumers' intrinsic desire for agency, allowing them to feel a sense of control and participation that traditional pricing models cannot provide [14]. The interactive nature triggers positive emotional responses, potentially enhancing satisfaction and encouraging repeat engagement.

Cultural contexts significantly shape consumer receptivity to bargaining mechanisms. In markets where haggling is culturally prevalent, consumers may readily embrace algorithmic bargaining, while regions accustomed to fixed pricing might initially approach such systems with skepticism. This variability underscores the importance of contextual design and personalized negotiation strategies that can accommodate diverse consumer expectations and psychological needs.

The complexity of consumer behavioral responses is further nuanced by individual factors such as risk tolerance, digital literacy, and technological exposure. Younger, more technologically integrated demographics tend to exhibit greater comfort with innovative pricing mechanisms, whereas traditional consumer segments may approach these systems more cautiously. This differential response highlights the need for adaptive implementation strategies that can bridge technological innovation with user comfort.

From a psychological perspective, algorithmic bargaining taps into fundamental reward-seeking behaviors. The process of negotiation and potential achievement of a favorable outcome triggers positive emotional responses like satisfaction and excitement. By integrating principles of variable rewards, these systems can sustain consumer interest and create a more engaging transactional experience that goes beyond mere price determination.

Empirical observations suggest that the success of algorithmic bargaining hinges on its ability to balance technological sophistication with user-centric design. Platforms must carefully consider interface design, communication strategies, and the underlying computational logic to create systems that feel both fair and accessible. The interplay between technological capability and human perception becomes crucial in determining the ultimate effectiveness of these innovative pricing approaches.

7. Challenges and Ethical Considerations

Algorithmic bargaining presents a multifaceted array of ethical challenges, particularly concerning fairness, transparency, accessibility, and privacy in pricing practices. Fairness remains a cornerstone concern, as algorithms, while ostensibly neutral, risk perpetuating societal biases embedded within their training data. Historical pricing patterns reflecting discriminatory practices may inadvertently be replicated or amplified by these algorithms, exacerbating existing inequities. Addressing this requires meticulous data curation, ongoing monitoring, and embedding fairness constraints into the algorithms themselves. Moreover, understanding the diverse perceptions of fairness across consumer groups is crucial for fostering trust and equity. Clear metrics for assessing fairness, coupled with mechanisms for redress in cases of unjust outcomes, are essential to ensure responsible implementation [3].

Transparency in algorithmic pricing is critical for ethical operations and consumer trust. However, the complexity of machine learning models often renders their decision-making processes opaque, raising concerns about manipulation and exploitation. Simplifying explanations of algorithmic logic, allowing users to simulate pricing scenarios, or developing standardized frameworks for algorithmic transparency can alleviate consumer anxieties. At the same time, the challenge of safeguarding proprietary information while maintaining sufficient transparency highlights a delicate balance. Platforms must navigate this tension carefully, ensuring that disclosures promote understanding without inviting manipulation or undermining competitive advantages.

Accessibility is another significant concern, as complex pricing mechanisms can disproportionately disadvantage consumers with limited digital literacy or access to advanced technologies. Intuitive and inclusive interfaces that adapt to varying user needs are paramount [9]. This may include text-based, voice-based, or visual interaction options, adherence to accessibility guidelines, and compatibility with assistive technologies. Designing with inclusivity in mind ensures that the benefits of algorithmic bargaining are equitably distributed across diverse user demographics.

Privacy considerations are particularly salient in algorithmic bargaining, which depends on extensive data collection to personalize pricing. Consumers must be afforded control over their data, with clear explanations of its collection, usage, and storage. Robust consent mechanisms, stringent data security practices, and transparency in data governance are vital to maintaining trust. Techniques like federated learning and other privacy-preserving methods offer promising pathways for balancing data utility with privacy. Ultimately, platforms must reconcile the tension between personalization and privacy by prioritizing consumer autonomy while sustaining the operational effectiveness of dynamic pricing models.

Algorithmic manipulation presents a subtle yet profound ethical challenge. Sophisticated systems designed to optimize outcomes may exploit psychological vulnerabilities [1], blurring the line between personalization and predatory pricing practices. Protecting consumers from such exploitation necessitates vigilant oversight of algorithmic design and implementation [15]. Interdisciplinary collaborations involving technologists, ethicists, and regulatory authorities are vital to developing robust frameworks that safeguard consumer interests while enabling technological innovation. The evolving regulatory landscape further compounds these challenges, requiring adaptive legal frameworks that reconcile consumer protections with the dynamic nature of algorithmic advancements. Balancing these considerations is essential for fostering an equitable and ethical algorithmic bargaining ecosystem.

8. Regulatory and Market Implications

The implementation of algorithmic bargaining introduces significant regulatory challenges, necessitating a reevaluation of existing legal frameworks for digital marketplaces. Regulatory bodies face the intricate task of aligning policies with rapidly evolving technological mechanisms that underpin dynamic pricing systems. The adaptive nature of these algorithms, which learn from real-time data and evolve continuously, calls for regulatory approaches that are both flexible and responsive to innovation. Policymakers must possess a nuanced understanding of these technologies to anticipate potential risks and address their implications on market dynamics effectively. Developing agile and robust regulatory structures capable of adapting to unforeseen technological advancements is critical for balancing innovation with consumer protection.

The widespread adoption of algorithmic bargaining has profound implications for competitive market dynamics. These systems provide platforms with significant competitive advantages by enhancing pricing flexibility, fostering personalized consumer engagement, and enabling real-time responses to market fluctuations [7]. The ability to implement data-driven, targeted pricing strategies allows platforms to optimize revenue generation and differentiate themselves in increasingly competitive markets. However, this raises concerns about the potential consolidation of market power among larger players with greater access to data and advanced computational resources [11]. Smaller platforms and new entrants may find it challenging to compete, exacerbating existing market inequalities. Investigating the long-term effects of algorithmic bargaining on market concentration is essential for ensuring a competitive and inclusive marketplace.

Antitrust considerations play a pivotal role in the regulatory discourse surrounding algorithmic bargaining [16]. The sophisticated nature of these technologies creates opportunities for anti-competitive practices, such as implicit collusion or market manipulation, even without direct human intervention. Algorithms may inadvertently coordinate pricing strategies, effectively fixing prices across competing platforms. Detecting and mitigating such behaviors require advanced computational forensic capabilities and stringent regulatory oversight. Crafting clear guidelines that distinguish between legitimate dynamic pricing practices and anti-competitive manipulations is vital to maintaining the integrity of open markets while fostering innovation.

Market entry barriers are another critical consideration in the context of algorithmic bargaining. The deployment of advanced pricing algorithms often requires substantial financial and technical resources [4], potentially creating a technological divide between established platforms and smaller players. This disparity risks the formation of oligopolistic market structures where dominant platforms monopolize the benefits of algorithmic bargaining. Regulatory interventions, such as promoting open-source algorithmic tools or establishing equitable data-sharing initiatives, may be necessary to mitigate these disparities and ensure fair competition. Such measures can empower smaller platforms, enabling them to compete effectively in a landscape increasingly shaped by sophisticated pricing mechanisms.

Data governance and privacy concerns are central to the regulatory challenges of algorithmic bargaining. The extensive reliance on consumer data for personalized pricing necessitates robust legal frameworks that prioritize transparency and user control. Regulatory approaches must ensure that platforms implement clear and accessible data collection practices, secure informed consent from users, and establish accountability mechanisms for breaches or misuse. The cross-border nature of many digital platforms further complicates data governance, requiring international cooperation to harmonize privacy regulations and establish consistent standards. The exploration of privacy-preserving technologies, such as federated learning, represents a promising avenue for reconciling innovation with consumer autonomy in algorithmic bargaining.

Economic policy frameworks must adapt to the complexities introduced by algorithmic bargaining. Traditional models based on fixed or predictable pricing fail to capture the dynamic and personalized nature of these systems. Policymakers must develop innovative analytical tools to assess the impact of algorithmic pricing on consumer welfare, market behavior, and overall economic stability. Interdisciplinary collaboration among economists, computer scientists, and legal scholars is essential for bridging the gap between technological advancements and policy considerations. The potential for algorithmic pricing to exacerbate economic inequalities further underscores the importance of equitable policy interventions that promote access to goods and services across diverse demographic groups.

The global nature of digital marketplaces adds a layer of complexity to the regulatory landscape. Platforms operating across multiple jurisdictions face varying legal standards for dynamic pricing and data privacy. Achieving international regulatory harmonization is crucial for fostering innovation while ensuring consistent consumer protection. Collaborative efforts among regulatory bodies, industry stakeholders, and consumer advocacy groups are essential for developing common standards and best practices. Establishing cross-border dialogue and knowledge-sharing mechanisms can facilitate the creation of a stable and predictable global regulatory framework for algorithmic bargaining systems. This global approach is essential to addressing the multifaceted challenges posed by the adoption of algorithmic pricing on a transnational scale.

9. Comparative Case Studies

Existing literature offers limited empirical evidence on real-world implementations of algorithmic bargaining, especially concerning how consumers react to and engage with these systems. However, we can analyze related examples of dynamic pricing and algorithmic decision-making to draw comparisons and infer potential implications. Consider the ride-sharing platform Uber, which utilizes a dynamic pricing model based on real-time supply and demand. While not strictly bargaining, this algorithmic system adjusts prices based on passenger behavior, such as surge pricing during peak hours or events. Passengers, aware of this mechanism, may strategically alter their behavior, delaying trips or seeking alternative transportation, which mirrors the strategic avoidance observed in. This dynamic interaction, although driven by fluctuating market conditions rather than explicit bargaining, demonstrates consumers' capacity to respond strategically to algorithmic pricing mechanisms. Similarly, e-commerce platforms like Amazon use algorithms to personalize product recommendations and potentially adjust pricing based on browsing history and purchase patterns. Although the exact workings of these algorithms remain opaque, consumers may perceive them as personalized negotiations, influencing their purchasing decisions.

Further insights can be drawn from research on consumer reactions to algorithmic decision-making in other contexts [15] demonstrate that consumers react less positively to favorable decisions (e.g., loan approvals) made by algorithms compared to humans, while the difference is mitigated for unfavorable decisions. This suggests that in bargaining scenarios, a perceived lack of human involvement might negatively affect consumer satisfaction, especially when offered a less advantageous deal by the algorithm which highlights the potential role of social preferences in algorithmic bargaining, suggesting that consumers might be more receptive to algorithmic offers if they perceive a social benefit or a reduced sense of direct exploitation by a non-human entity.

Finally, the increasing use of AI-powered chatbots for customer service and support opens a new avenue for exploring algorithmic bargaining in real-world settings. These chatbots, often designed to mimic human conversation, could be programmed to negotiate prices or offer personalized deals based on customer interactions. This could potentially provide a more engaging and interactive bargaining experience, potentially mitigating some of the negative perceptions associated with impersonal algorithms. However, the potential for manipulation and the ethical implications of such systems must be carefully considered.

10. Future Directions in Algorithmic Bargaining

The progression of algorithmic bargaining is closely tied to advancements in technology and interdisciplinary research, presenting numerous avenues for future development. Blockchain technology emerges as a transformative tool for enhancing transparency and trust in dynamic pricing mechanisms. With its decentralized and immutable architecture, blockchain can address long-standing concerns about pricing opacity and manipulation. By creating verifiable transaction records, blockchain fosters consumer confidence and ensures accountability in algorithmic processes. This technology could enable innovative applications, such as smart contracts that automatically execute transactions based on negotiated prices, driving both fairness and efficiency. Future research must address the challenges of integrating blockchain with algorithmic bargaining systems, focusing on scalability, security, and regulatory compliance to guide its adoption in e-commerce and beyond.

Advanced machine learning algorithms are poised to redefine algorithmic bargaining by enabling highly personalized and adaptive pricing strategies. Techniques such as neural networks and deep learning models can analyze complex relationships between user behavior, market dynamics, and pricing strategies. These models facilitate near-instantaneous pricing adjustments that anticipate consumer preferences with unprecedented precision, enhancing user experience and platform revenues. However, the sophistication of these systems necessitates research into interpretability and fairness, as concerns over algorithmic bias and explainability grow. The development of transparent machine learning models and methods for bias mitigation will be critical for fostering trust in these systems. Reinforcement learning presents another promising avenue, optimizing bargaining strategies in dynamic market environments and offering robust, real-time adaptability.

The continued evolution of algorithmic bargaining aligns with Moore's Law, which predicts exponential growth in computational power. This trend enables the processing of vast datasets with minimal latency, incorporating multidimensional consumer insights and real-time market data into pricing models. As computational barriers diminish, the scalability and efficiency of these systems will increase, fostering broader adoption across industries. Research focusing on the scalability of algorithmic bargaining systems, particularly in high-volume environments, is essential for maintaining robust performance. Additionally, exploring edge computing could enhance the responsiveness and efficiency of these systems, enabling faster decision-making at the point of interaction.

The potential applications of algorithmic bargaining extend far beyond e-commerce, with significant opportunities in service industries, financial markets, real estate, insurance, and global trading platforms. Each of these sectors presents unique challenges that require tailored algorithmic solutions and regulatory frameworks. For instance, in financial markets, algorithms must comply with stringent transparency and fairness requirements while addressing systemic risks and potential manipulation. As the scope of adoption widens, research must address the specific technical and ethical considerations for implementing these systems in diverse contexts, ensuring their adaptability and reliability.

Interdisciplinary research will play a pivotal role in addressing the broader implications of algorithmic bargaining. Collaboration between computer scientists, economists, psychologists, and legal scholars is essential for developing pricing mechanisms that are efficient, ethical, and culturally sensitive. Understanding the psychological effects of algorithmic bargaining on consumer behavior and addressing potential exploitation are key research priorities. Additionally, investigating cultural variations in consumer perceptions and acceptance of bargaining practices will be crucial for designing globally applicable solutions.

Frontier technologies, including artificial intelligence and quantum computing, hold immense promise for the future of algorithmic bargaining. Quantum algorithms, for example, could revolutionize dynamic pricing by enabling the simultaneous evaluation of numerous pricing scenarios, leading to unprecedented levels of responsiveness and precision. Integration of real-time global economic indicators, geopolitical developments, and instantaneous market sentiment analysis could elevate these systems into sophisticated economic interaction platforms. Research exploring the feasibility and scalability of quantum computing in pricing optimization, as well as its ability to process diverse data sources, remains in its infancy but is critical for advancing this field.

The trajectory of algorithmic bargaining will be shaped by the convergence of technological advancements and interdisciplinary insights. By leveraging blockchain for trust, machine learning for adaptability, and computational advancements under Moore's Law, these systems will redefine dynamic pricing practices across industries, paving the way for innovative and responsible implementations.

11. Conclusion

This research examined the innovative paradigm of algorithmic bargaining within dynamic pricing systems, focusing on its usability, technological underpinnings, consumer experiences, ethical challenges, and market implications. Algorithmic bargaining, characterized by interactive negotiation capabilities, demonstrates potential to revolutionize customer engagement and tailor online shopping experiences. However, it also introduces complexities including the technical intricacies of real-time algorithm deployment, challenges in achieving fairness and transparency, privacy considerations, and the uncertainties of regulatory frameworks. These issues necessitate a comprehensive and multidisciplinary approach to implementation and oversight.

Key findings underscore the criticality of transparency in pricing mechanisms to foster consumer trust and diminish perceptions of exploitation. Equally important is the development of user-friendly interfaces that enable inclusive participation across diverse consumer groups, mitigating risks of digital exclusion. Ethical concerns, particularly regarding discriminatory pricing practices, demand the establishment of robust guidelines alongside adaptive regulatory frameworks. Furthermore, the success of these systems depends on iterative algorithmic refinement to adapt to dynamic market behaviors and minimize biases embedded within data sources.

Empirical analyses and comparative evaluations of existing dynamic pricing models, though distinct from algorithmic bargaining, provided valuable contextual insights. These studies emphasized the importance of culturally attuned and personalized approaches to pricing strategies, highlighting variations in consumer expectations and market behaviors across global contexts.

Future research should prioritize the integration of emerging technologies such as blockchain to bolster transparency and enhance trust in algorithmic systems. The development of sophisticated machine learning techniques for more nuanced and ethical personalization of pricing strategies presents a promising avenue. Additionally, expanding the scope of algorithmic bargaining to sectors beyond e-commerce such as healthcare, transportation, and utilities could offer transformative societal benefits. Addressing these challenges requires collaborative efforts among technologists, economists, legal scholars, and behavioral scientists to navigate the intersection of technical innovation and societal impact.

As algorithmic bargaining matures, its responsible deployment will necessitate a balanced approach that aligns technological innovation with ethical imperatives. The continuous engagement of industry stakeholders, regulatory bodies, and consumer advocates will be vital to establishing a fair and equitable digital marketplace. Through such a collaborative and adaptive framework, algorithmic bargaining has the potential to reshape dynamic pricing paradigms while safeguarding consumer welfare and trust.

REFERENCES

1. E. Cartwrigh, Behavioral Economics, Abingdon, Oxfordshire: Routledge, 2011.
2. J. Haessner, P. Haessner and M. McMurtrey, "Dynamic Pricing in Different Industries," *Journal of Marketing Development and Competitiveness,* 2023.
3. S. Martin, M. Bertini, O. Koenigsberg, R. Zeithammer, D. Aparicio, Y. Chen, F. Fantini, G. Z. Jin, V. Morwitz, P. P. Leszczyc, M. A. Vitorino, G. Y. Williams and H. Yoo, "ALGORITHMIC PRICING: IMPLICATIONS FOR CONSUMERS, MANAGERS, AND REGULATORS," NATIONAL BUREAU OF ECONOMIC RESEARCH, Cambridge, 2024.
4. R. P. McAfee and V. t. Velde, "Dynamic Pricing in the Airline Industry," California Institute of Technology, Pasadena.
5. T. Heiskanen and B. Friedland, "Upgrade to AI-driven dynamic pricing," 27 October 2023. [Online]. Available: https://www.thoughtworks.com/insights/blog/machine-learning-and-ai/upgrade-to-AI-driven-dynamic-pricing.
6. M. Backus, T. Blake, B. Larsen and S. Tadelis, "Sequential Bargaining in the Field: Evidence from Millions of Online Bargaining Interactions," *The Quarterly Journal of Economics,* p. 1319–1361, August 2020.
7. N. Chen and G. Gallego, "Do Consumers Benefit from Dynamic Pricing?," *SSRN Electronic Journal,* January 2016.
8. A. L. Smoulder, N. P. Pavlovsky, P. J. Marino, A. D. Degenhart, N. T. McClain, A. P. Batista and S. M. Chase, "Monkeys exhibit a paradoxical decrease in performance in high-stakes scenarios," *Proceedings of the National Academy of Sciences,* 31 August 2021.
9. J. Nielsen, Usability Engineering, San Francisco, CA: Morgan Kaufmann Publishers Inc., 1993.
10. C. M. Bishop, Pattern Recognition and Machine Learning, New York, NY: Springer Science+Business Media, LLC, 2006.
11. T. H. Davenport, "Competing on Analytics," *Harvard Business Review,* pp. 98-107, February 2006.
12. C. O'Neil, Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy, New York: Crown Publishers, 2016.
13. D. Kahneman, J. L. Knetsch and R. Thaler, "Fairness as a Constraint on Profit Seeking: Entitlements in the Market," *The American Economic Review,* vol. 76, pp. 728-741, September 1986.
14. E. L. Deci and R. M. Ryan, "The "What" and "Why" of Goal Pursuits: Human Needs and the Self-Determination of Behavior," *Psychological Inquiry,* vol. 11, pp. 227-268, 2000.
15. G. Yalcin, S. Lim, S. Puntoni and S. M. v. Osselaer, "Thumbs Up or Down: Consumer Reactions to Decisions by Algorithms Versus Humans," *Journal of Marketing Research,* 2022.
16. C. Lee, "The Landscape of Pricing and Algorithmic Pricing," *ISEAS-Yusof Ishak Institute,* 2020.

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PERFORMANCE BENCHMARKING OF EMBEDDED EDGE DEVICES FOR VARIOUS FACE RECOGNITION MODELS

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ABSTRACT

Artificial Intelligence (AI) models are increasingly pivotal in enabling face recognition across various fields, from educational and research settings to public spaces. Effective deployment of these models requires high-performance hardware, such as RTX graphics cards or embedded edge devices like Nvidia's AGX Orin and Jetson Nano. This paper presents a comprehensive benchmarking study comparing the performance of these two devices, representing high and low-power edge computing options, using two face recognition models: ResNet and MobileNet.
The benchmarking process assesses each model across two different input sizes deployed on both devices with varied configurations, including CPU thread allocation and GPU power distribution within containerized environments. Performance metrics such as inference time, GPU utilization, memory usage, and CPU load are analyzed to determine each device's suitability and efficiency. Additionally, model-specific parameters, including FLOPS, parameter count, and memory footprint, are examined to provide for an in-depth comparison. This paper presents detailed results and analyses of these performance indicators.

I. Introduction

Face recognition technology, driven by advancements in artificial intelligence (AI), has become essential in numerous applications, including public safety, education, and retail. With the increasing demand for real-time, low-latency processing, deploying AI models on embedded edge devices has emerged as a viable solution, offering benefits like localized data processing and reduced dependency on cloud infrastructure. Among edge computing solutions, NVIDIA's Jetson series provides popular choices such as the Jetson AGX Orin [5] and Jetson Nano [6], each representing different power and performance spectrum ends. As Table 1 shows, the AGX Orin, with its high computational capabilities, is designed for intensive AI tasks, while the Jetson Nano provides a more compact, low-power option suitable for lighter, localized applications.

Nvidia Jetson Module	Jetson AGX Orin	Jetson Nano
CPU	8-core ARM Cortex-A78AE @ 2.188 GHz	4-core ARM Cortex-A57 @ 1.43GHz
GPU	NVIDIA Ampere architecture GPU with 2048 CUDA cores	128-core Maxwell GPU
Memory	32 GB LPDDR5	4 GB LPDDR4
Storage	64 GB eMMC	No onboard storage (microSD)
Storage Type	eMMC 5.1 + NVME	microSD only
Power	15-60 W (configurable)	5-10 W
JetPack	JetPack 6.0	JetPack 4.2.1
Framework	TensorFlow, PyTorch	TensorFlow, PyTorch
AI Performance	Up to 275 TOPS	0.5 TOPS

Table 1: Specifications of Jetson boards

This paper aims to evaluate the performance of these two devices in handling face recognition tasks by benchmarking two well-known deep learning models, ResNet and Mobile Net. These models are widely used in computer vision for their accuracy and efficiency, making them ideal candidates for deployment in edge environments with limited resources. To further simulate real-world usage, this study uses WIDER Face and FDDB datasets, which contain challenging and varied face images across different environments, poses, and lighting conditions. This dataset choice allows for an in-depth assessment of each model's robustness and suitability for deployment on edge devices.

The benchmarking examines vital performance metrics, including inference time, GPU utilization, memory usage, and CPU load. Both devices are tested with different input sizes and configurations, such as CPU thread allocation and GPU power settings within edge devices.

Our study makes several contributions:

− A detailed comparative analysis of the Jetson AGX Orin and Jetson Nano devices for face recognition applications, focusing on how each handles resource-intensive tasks.

− Performance benchmarks across two AI models and two datasets offer insights into which model-hardware combinations are optimal for different face recognition scenarios.

− Exploring resource management, including model optimization and hardware scaling, impacts performance, providing valuable guidance for deploying AI on edge devices effectively.

This research addresses the growing need to understand how edge device configurations impact the performance of complex AI models. By examining these aspects, we offer a guide to optimizing face recognition model deployment on edge devices in practical settings where balancing efficiency and accuracy is critical.

II. Related Work

In the last few years, there have been several AI deployed benchmarking tests performed using Deep Neural Networks (DNNs) on different Nvidia Jetson Devices. The famous papers and their contributions are given below:

(i) DNN on embedded boards:

A study in [1] analyzed the performance of three edge devices - NVIDIA Jetson Nano, TX2, and Raspberry Pi 4 by running a convolutional neural network (CNN) model designed to classify fashion products. The comparison considered factors like power consumption, resource utilization (GPU, CPU, RAM), model accuracy, and overall cost, using datasets between 5,000 and 45,000 images. Relatively in [2] the authors introduced EdgeFace, an face recognition model designed specifically for edge devices, which reduces computational overhead while maintaining high accuracy.

(ii) Benchmarking DNNs on Jetson boards:

Evaluating Deep Learning on Jetson Devices: Jetson boards have been widely tested for deep learning deployment. In [3], researchers assessed 3D point cloud classification across various dataset sizes, analyzing computational demands on multiple Jetson models. Another study in [4] compared the performance of 3D object detection models specifically on NVIDIA Jetson AGX Xavier and Nano, examining their efficiency and processing capabilities.

III. Benchmark Analysis

A. Experimental setup

We benchmarked two Jetson devices: the AGX Orin as the high-performance setup and the

Nano as the lower-performance setup. Figure 1 illustrates the use of ResNet and MobileNet models for the Face Recognition algorithm, utilizing the Face Detection Dataset and Benchmark (FDDB) and Wider Face datasets. For these tests, we employed two data resolutions per dataset: 320x320 as the lightweight configuration and 800x800 as the heavy configuration for FDDB, and similarly, 224x224 as the lightweight and 640x640 as the heavy configuration for Wider Face.

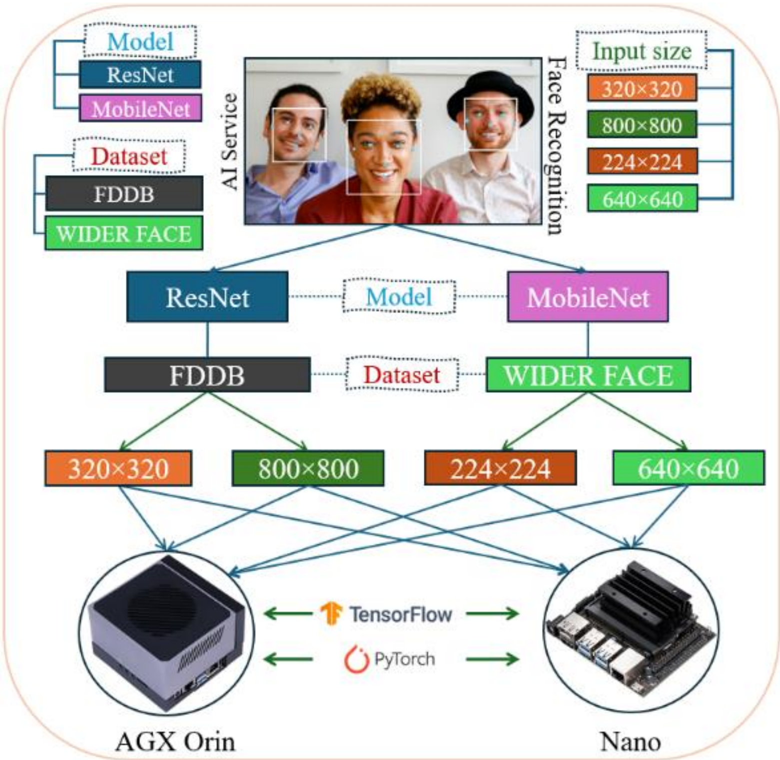

Figure 1. Architecture of Embedded Edge Device and AI Model Deployment

This allows us to assess how each embedded edge device manages various workloads and data complexities. All platforms were configured to maximize CPU performance by utilizing all available cores at their highest frequency, ensuring optimal performance levels.

B. Benchmarking results

We benchmarked the performance of Face Recognition using two models, ResNet and MobileNet, across FDDB and Wider Face datasets with varying input sizes on both the AGX Orin (high-performance setup) and Nano (low-performance setup).

The results, detailing inference time, memory usage, and power consumption, are presented in Table 2.

Algorithm	Model	Dataset	Input size	Inference Time (ms)		Memory (G)		CPU (Power/mW)		GPU (Power/mW)	
				AGX Orin	Nano	AGX Orin	Nano	AGX Orin	Nano	AGX Orin	Nano
Face Recognition	ResNet	FDDB	320x320	12	45	2.0	1.5	350	700	900	350
			800x800	28	95	3.2	2.2	400	800	1100	500
	Mobile Net	Wider Face	224x224	7	25	1.5	1.0	300	600	700	250
			640x640	22	70	2.3	1.8	350	650	850	400

Table 2. Benchmarking results

Inference times varied significantly based on model and input size. For ResNet on FDDB, inference times ranged from 12 ms to 28 ms on AGX Orin, while Nano required between 45 ms and 95 ms. MobileNet on the Wider Face dataset showed faster performance, with AGX Orin achieving 7 ms to 22 ms, and Nano taking between 25 ms and 70 ms.

Memory consumption scaled predictably with input size and model complexity. ResNet on FDDB required up to 3.2G on AGX Orin, while Nano used up to 2.2G. MobileNet, being a more lightweight model, required less memory, with a maximum of 2.3G on AGX Orin and 1.8G on Nano for the larger input size.

Power consumption also demonstrated a clear pattern tied to model and hardware. The AGX Orin displayed higher power usage than the Nano, with CPU power ranging from 300 mW to 400 mW and GPU power up to 1100 mW, depending on the model and input size. In contrast, Nano's CPU and GPU power demands were significantly lower, with CPU power peaking at 800 mW and GPU power at 500 mW.

Overall, AGX Orin consistently provided faster inference times and supported higher memory and power resources, making it more suitable for computationally intensive face recognition tasks. The Nano, though slower, offered a more energy-efficient alternative for less demanding applications, illustrating a trade-off between computational power and energy efficiency.

Conclusion

In this paper, we conducted a benchmarking study of two different Face Recognition models on two Nvidia Jetson devices: the Jetson AGX Orin and the Jetson Nano. Our goal was to assess the performance of these platforms across multiple datasets and input sizes using a Face Detection algorithm. Both devices were configured for maximum performance to achieve higher processing power. Throughout the tests with varying input sizes, we measured memory usage, CPU and GPU utilization, and overall power consumption.

Our experiments revealed that the AGX Orin consistently outperformed the Nano in terms of inference speed, particularly with more complex models such as ResNet and MobileNet. However, Nano demonstrated superior power efficiency and resource management when running simpler models with lightweight datasets.

REFERENCES

[1] Süzen, A.A.; Duman, B.; Şen, B. Benchmark Analysis of Jetson TX2, Jetson Nano and Raspberry PI using Deep CNN. In Proceedings of the 2020 International Congress on Human-Computer Interaction, Optimization and Robotic Applications (HORA), Ankara, Turkey, 26–28 June 2020; pp. 1–5.

[2] A. George, C. Ecabert, H. O. Shahreza, K. Kotwal and S. Marcel, "EdgeFace: Efficient face recognition model for edge devices", IEEE Trans. Biometrics Behav. Identity Sci., vol. 6, no. 2, pp. 158-168, Apr. 2024.

[3] S. Ullah and D.-H. Kim, "Benchmarking jetson platform for 3d point cloud and hyper-spectral image classification," in 2020 IEEE International Conference on Big Data and Smart Computing (BigComp). IEEE, 2020, pp. 477–482.

[4] M. Choe, S. Lee, N. M. Sung, S. Jung and C. Choe, "Benchmark Analysis of Deep Learning-based 3D Object Detectors on NVIDIA Jetson Platforms", International Conference on ICT Convergence IEEE Computer Society, pp. 10-12, 2021.

[5] NVIDIA: AGX Orin: https://www.nvidia.com/en-us/autonomous-machines/embedded-systems/jetson-orin/

[6] NVIDIA: Jetson Nano: https://developer.nvidia.com/embedded/jetson-nanodeveloper-kit

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DEVELOPMENT OF CONTROL ALGORITHM FOR FLEXIBLE MANUFACTURING SYSTEM MODULE USING FİNİTE AUTOMATA WİTH PETRİ NET

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ABSTRACT

On the basis of machine-building industry area analysis there were defined actuality and a goal of this material. As the applied object of this study, flexible manufacturing system is chosen and also it was given a task of designing an algorithm for investigation of its control system. When creating a control model for a flexible manufacturing system a finite automata was used to describe the operation of a crane manipulator. The capabilities of the finite automata are not enough for a general system analysis of the control object. For a detailed functional analysis with the definition of transfers and productions of this control object, it is necessary to use a more powerful mathematical apparatus, such as Petri net. The study of the control algorithm was held on the basis of the round composed flexible manufacturing module scheme of flexible manufacturing system. The resulting algorithm with using the transformation of a finite automata of Petri net allows for the creation of a more correct, efficient and reliable control system for the production module.

INTRODUCTION

In modern times, one of the areas that ensure the economic development of the leading Eurasian countries is considered to be industrial enterprises producing high-quality mechanical products of mechanical engineering. In flexible manufacturing system (FMS), which combine mechanical assembly, processing and production modules of various purposes, automation of the process of manufacturing various electrical, automatic, mechatronic, etc. parts for technological equipment, machine tools and production lines is carried out [1,2]. However, since many complex mechanical technological operations are performed both by working personnel and by control units controlled by digital programs, the design of both mechanical machines and a special manipulator that automatically serves them in the modules of mechanical FMSs of mechanical engineering enterprises and the creation of a control system for automating its operations is considered a scientifically relevant issue.

APPLİCATİON AREA ANALYSİS

The layout of the mechanical production module of the applied FMS is based on a circular structure (Fig. 1).

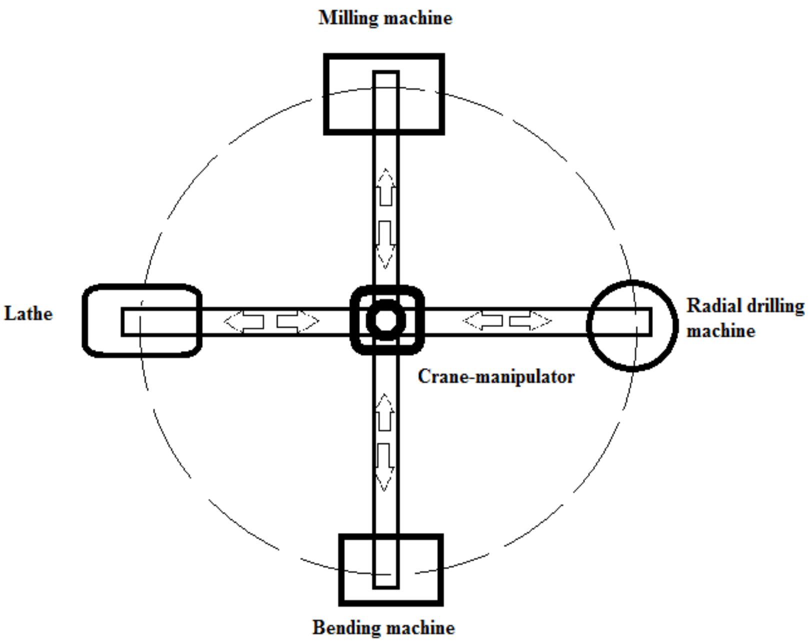

Fig. 1 Layout diagram of the mechanical production module of the FMS

In order to reliably control the service operations of lathes, milling machines, radial drills and bending machines located in the positions of the circular scheme, the issue of conducting a functional analysis of the technological process and interpreting the control functions with knowledge is posed. The analysis of this problem shows that it is more expedient to apply a graphical representation method to more effectively describe the control process [3].

In the mechanical flexible manufacturing module of the FMS, technological operations are planned as follows: the crane-manipulator takes the workpiece from the central plate and moves it linearly upwards. Then, moving linearly on the crane in the left direction, it positions itself in the working zone of the lathe, places the workpiece on the machine, fixes it and moves upwards, waiting for the preparation of the part [4]. After the product is ready, the manipulator moves linearly downwards and takes the finished part from the lathe and moves linearly upwards. After linear movement in the central direction, it moves the part in the direction of the milling or radial drilling machine and the bending machine. The sequence of these operations depends on the type of product being prepared [5].

PROBLEM STATEMENT AND DESCRİPTİON

The One of the more accurate and reliable methods for describing the planning and functions of a complex machine-building technological process control system is considered to be a Petri net (PN).

Modeling of complex dynamic systems (DS) can be considered at several levels, and PNs can model each of these levels. But in practice, it is often the case that individual subtasks of a complex system are modeled using finite automata (FA). Due to the simplicity of application at a low level, system modules are often described with FA. The FA apparatus is intended primarily for modeling individual processes. When modeling complex DS, it is necessary to take into account not only individual processes, but also their interaction. In particular cases, such problems are solved by the capabilities of the FA themselves. This is accomplished by connecting the outputs of some machines to the inputs of others.

At a high level, models based on PN are more convenient for providing a general system analysis of the control object. For formal verification of the system when modeling complex dynamic systems, it is necessary to use a sufficiently powerful formal apparatus such as PN. In this case, there is often a need to solve the emerging problem of transforming the FA into PN.

To describe the control process using a PN, it is possible to effectively describe the operation of the crane manipulator (CM) serving the application object (mechanical assembly flexible manufacturing module) with production rules and, having transformed it into a PN, study the control algorithm using computer experiments as a result of the analysis of the main properties of the latter. This issue was considered in the previous subsection [6].

The issues of developing an algorithm for transforming the finite automata representation of an automatic transport manipulator transporting raw materials to the working area of a crane manipulator into a PN are considered.

Finite automata (FA) is defined by a set of six objects:

$$A = (X, U, Y, x_0, \varphi, \psi)$$

Where $U = (u_1, u_2, \dots, u_m)$ – a finite set of input signals, called the input alphabet of the automata; $Y = (y_1, y_2, \dots, y_g)$ – a finite set of output signals, called the output alphabet of the automata; $X = (x_1, x_2, \dots, x_z)$ – an arbitrary set, called the set of internal states of the automata; x_0 – an element from the set X called the initial state of the automata; $\varphi: (x, u)$ and $\psi: (x, u)$– two functions that define singular-valued maps of the set of pairs (x, u), where $u \in U$ and $x \in X$, to the sets X and Y. The function $\varphi: (x, u)$ is called the transition function of the automata, and the function $\psi: (x, u)$ is called the output function [7]:

$$\varphi: (X \times U) \rightarrow X; \ \psi: (X \times U) \rightarrow Y.$$

FA can be represented graphically, in the form of transition and output tables, and analytically. $\varphi: (x, u)$ and $\psi: (x, u)$the transition and output functions of the automata can be represented in the form of a table.

A PN is formally represented as a set of the form [8]:

$$N = (P, T, F, H, \mu^0)$$

Where:

$P = \{p_1, p_2, \dots, p_n\}$, $n > 0$ – *a* finite non-empty set of positions (otherwise states or locations);

$T = \{t_1, t_2, \dots, t_m\}$, $m > 0$ – a finite non-empty set of transitions (events);

$F: P \times T \rightarrow \{0,1,2, \dots\}$ and $H: P \times T \rightarrow \{0,1,2, \dots\}$ - the initial marking (marking of each position) of the functions and mapping of the input and output states;

$\mu^0: P \rightarrow \{0,1,2, \dots\}$ – initial distribution of markers by position.

The graphical representation of a PN is a bidirectional directed graph with two types of vertices. The vertices p ∈ P are represented by circles, and the vertices t ∈ T are represented by with rectangles. The arcs correspond to the drop functions of positions and transitions .

The FA is defined for the PN as follows:

$$P = U \cup X \cup Y, T = \{t_x, x / x \in X \text{ и } u \in U\},$$

$$I(t_x, u) = \{x, u\}, O(t_x, u) = \{\varphi(x, u), \psi(x, u)\}.$$

The resulting PN is a model of a FA.

Based on the given topology, the definition of the structural elements of the PN is presented - transitions and positions according to the following rules [8]: for each pair (state and input symbol), a transition is defined; the combination of sets of input signals $U = (u_1, u_2, \dots, u_m)$, output signals $Y = (y_1, y_2, \dots, y_g)$, internal states $X = (x_1, x_2, \dots, x_z)$ is a set of positions of the automata. The transitions and positions of the PN are defined as $T = \{t_1, t_2, \dots, t_m\}$, , $P = \{p_1, p_2, \dots, p_n\}$, where $n = m + z + g$.

Let's consider the model of the functioning CM of the FMS. The CM can be in one of two states X=(x_1, x_2): x_1– open, - x_2– closed (number of internal states z=2). The transition from one state to another $U = (u_1, u_2)$ is carried out by control actions: u_1– closed, u_2– switching on the PN (number of input signals m=2). The transition and output functions are given by the following transformations [7]:

$$\begin{cases} \varphi: (x_2, u_1) \to x_1; \\ \varphi: (x_1, u_2) \to x_2; \end{cases} \quad (1)$$

$$\begin{cases} \psi: (x_2, u_1) \to y_1; \\ \psi: (x_1, u_2) \to y_2. \end{cases} \quad (2)$$

The external states of the CM are determined by the output alphabet: $Y = (y_1, y_2)$. $y_1$is off CM, y_2 is on CM (the number of output signals g=2). Therefore, to control the CM, it is necessary to determine the internal states X based on the information U from the corresponding sensors and generate control signals Y in accordance with the transition function.

Now, to define transitions from one state to another, we can represent the network transitions as follows. For each pair (state and input symbol), we define a transition whose input positions correspond to the state and input symbol, and whose output positions correspond to the next state and output.

Thus, $T = \{t_1, t_2\}$ the PN transitions are determined (number of transitions m=2). The PN positions (number of positions $n = m + z + g = 2 + 2 - 2 = 6$) are determined according to the following rule [8]:

$$P = U \cup X \cup Y = \{u_1, u_2, x_1, x_2, y_1, y_2\}$$

$$P = \{p_1, p_2, p_3, p_4, p_5, p_6\}.$$

CONSTRUCT A PN SIMULATING A FA

Creating a transition matrix:

$$C(m, z) = C(2,2) = \{c_{j,i}\}, i = \overline{1,2};\ k = \overline{1,2};\ j = \overline{1,2};$$

According to the (1):

$$c_{1,1} = 0, c_{1,2} = 1,\ c_{2,1} = 2, c_{2,2} = 0.$$

As a result, obtaining matrix transitions:

$$C(2,2) = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}.$$

Creating the output matrix:

$$B(m,z)=B(2,2)=\{b_{j,i}\},\ i=\overline{1,2};\ k=\overline{1,2};\ j=\overline{1,2};$$

According to the (2):

$$b_{1,1}=0,\ b_{1,2}=1, b_{2,1}=2, b_{2,2}=0.$$

Finally, the results matrix is obtained:

$$B(2,2)=\begin{vmatrix}0 & 1\\ 2 & 0\end{vmatrix}.$$

Determining the input incidence matrix of the Petri net:

$$F(n,m)=F(6,2)=\{f_{i,j}\},\ here\ i=\overline{1,6}; j=\overline{1,2};$$

$n=m+z+g=6$ (n-number of positions, m-number of transitions) .

Determining the output incidence matrix of the Petri net:

$$H(m,n)=H(2,6)=\{h_{j,i}\},\ here\ i=\overline{1,6}, j=\overline{1,2}.$$

Petri net entrance incidence matrix elements formation:

$$F\ (6,2)=\{f_{i,j}\}:$$

Matrix all elements resetting

$$f_{i,j}=0,\ here\ i=\overline{1,6}; j=\overline{1,2};$$

$$if\ c_{j,i}\neq 0,\ then\ f_{j,j}=1\ and\ f_{2+i,j}=1,$$

here $i=\overline{1,2}; j=\overline{1,2}$:

Because $c_{1,2}\neq 0$, it turns out $f_{j,j}=f_{1,1}=1$,

$$f_{2+i,j}=f_{2+2,1}=f_{4,1}=1;$$

Because $c_{2,1}\neq 0$, it turns out $f_{j,j}=f_{2,2}=1$,

$$f_{2+i,j}=f_{2+1,2}=f_{3,2}=1;$$

As a result, it is obtained that:

$$F(6,2)=\begin{vmatrix}1 & 0\\ 0 & 1\\ 0 & 1\\ 1 & 0\\ 0 & 0\\ 0 & 0\end{vmatrix}$$

Formation of the elements of the output events matrix $H(2,6)=\{h_{j,i}\}$:

Resetting all matrix elements to zero $h_{j,i}=0$, here $j=\overline{1,2};\ i=\overline{1,6}$;

if $c_{j,i}\neq 0$, then $h_{j,m+l}=h_{j,2+l}=1$, here $j=\overline{1,2};\ i=\overline{1,2}; l=c_{j,i}$;

Because $c_{1,2}\neq 0$, it turns out $h_{j,m+l}=h_{1,2+1}=h_{1,3}=1$, $(l=c_{j,i}=c_{1,2}=1)$;

Because $c_{2,1}\neq 0$, it turns out $h_{j,m+l}=h_{2,2+2}=h_{2,4}=1$, $(l=c_{j,i}=c_{2,1}=1)$;

Because $b_{j,i}\neq 0$, then $h_{j,m+z+l}=h_{j,2+2+l}=1$, here $j=\overline{1,2};\ i=\overline{1,2}; l=b_{j,i}$:

Because $b_{1,2} \neq 0$, it turns out that

$h_{j,m+z+l} = h_{1,2+2+1} = h_{1,5} = 1, (l = b_{j,i} = b_{1,2} = 1);$

Because $b_{2,1} \neq 0$, it turns out that

$h_{j,m+z+l} = h_{2,2+2+2} = h_{2,6} = 1, (l = b_{j,i} = b_{2,1} = 2);$

As a result, it is obtained that

$$H(2,6) = \begin{vmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{vmatrix} \quad (6)$$

Incident matrix elements of PN formation:

$$D(\mathrm{n,m}) = \mathrm{D}(6,2) = \{\mathrm{d_{i,j}}\}:$$

Because $D = F - H^T$, then $d_{i,j} = f_{i,j} - h_{j,i}$, here $j = \overline{1,2}$; $i = \overline{1,6}$;

$$D(6,2) = F(6,2) - H^T(2,6) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{vmatrix}.$$

$$D(6,2) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & -1 \end{vmatrix}.$$

Using the obtained results, a graph of a given finite automaton simulating Petri net was created in the CPN Tools system [8] (Fig. 2).

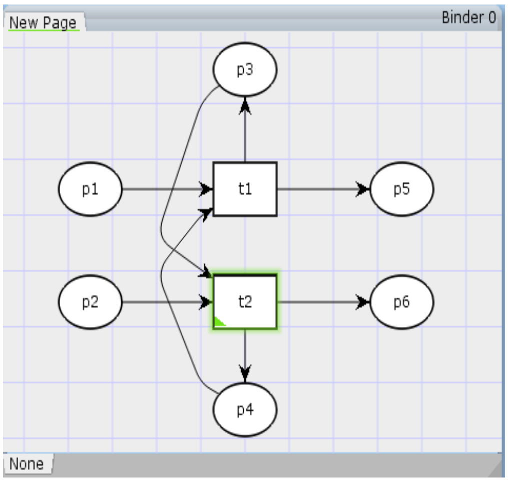

Fig. 2 Graph model of a PN simulating a FA given by transition and output functions.

CONCLUSION

The structure of the equipment of the FMS has been developed and descriptions of its control in the form of production rules and with FA have been created. The PN properties were used for a comprehensive analysis of the control of the computer model for FMS module described by a FA. The algorithm for transforming a FA into a PN was used for this. The model was studied as a result of simulation with computer experiments. The model of the control system of the agile manufacturing module obtained using FA is analyzed using PN. This approach can be considered important in terms of creating a more correct, effective and reliable control system.

REFERENCES

1. J. Mammadov, I. Aliyev, G. Huseynova & G. Orujova. (2021) Algorithmic Support for the Management of the Computer-Aided Design of Flexible Manufacturing System and its Equipment, Cybernetics and System Analyses, v. 57, p. 950-958.
2. I.R. Aliyev, J.F. Mammadov, S.R. Rakhimov. (2022) Selection of information-measuring components on the basis of layout diagram of flexible manufacturing cell, 11th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions and Artificial Intelligence-ICSCCW, p.124-131.
3. Mustafaev V.A., Huseynzade Sh.S. (2018) Development of a control model of a processing center using colored networks Petri, Bulletin of computer and information technologies, Moscow, 3(165), p. 36-44. (In Russian).
4. Kumar B., Mahesh V. (2015) Modeling and Analysis of Flexible Manufacturing System with FlexSim. International Journal of Computational Engineering Research (IJCER). 2250-3005, v. 05, Issue 10, p. 1-6.
5. Kolker A.B., Livenets D.A., Kosheleva A.I. (2012) Study of options for creating intelligent robotics systems based on single-board computers and free operating systems. Automation and Software Engineering, Novosibirsk. №.1, p. 84-98.
6. Mukhopad Y.F. (2014) Analysis and synthesis of control automata of complex technical systems, XII All-Russian meeting on control problems, Moscow, June 16-19, p. 7295-7306.
7. Huseynzade Sh.S. (2020) Transformation of finite automata tables into incident matrices of Petri net. Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications (COIA-2020). Baku, Azerbaijan, p. 173-176.
8. Akhmedov M.A., Huseynzade Sh.S., Nasirova E.A. (2019) Development of an algorithm for automating the transformation of a finite state machine into a Petri net". Automation. Modern technologies. Monthly inter-industry scientific and technological journal. Moscow, 2019T. 73, №3, p. 108-112. (In Russian)

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INVESTIGATION OF THE SECURITY OF A KEY EXCHANGE PROTOCOL BASED ON MATRIX ALGEBRA

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ABSTRACT

This paper considers the issue of assessing the security of a key exchange protocol based on matrix algebra. To protect the confidentiality of information, it is important that the keys are exchanged securely. This can be done by using methods such as encrypted key transmission, creation of dedicated secure channels, formation of a common key between parties without exchanging secret information using public key algorithms, etc. The paper analyzes the security of the protocol implemented using the approach based on non-invertible matrices against linear algebra and brute force attacks, shows that the use of non-invertible matrices increases the security of the system, and also evaluates the consumption of computing resources. The results confirm that this approach can be used as a secure and practical method of key exchange.

1. INTRODUCTION

The main problem of the symmetric encryption method, which is one of the most common methods of protecting the confidentiality of information, is the reliable delivery of the secret key to the parties. To solve this problem, various methods are used, such as sending the key in encrypted form, creating separate protected channels, delivering the key by courier, forming a common key between the parties without exchanging secret information using public-key algorithms, etc. The protocol created based on the Diffie-Hellman algorithm is a clear example of the public-key encryption method [1-2]. This algorithm allows two parties (sender and receiver) to create a common secret key using an open channel. The basis of the Diffie-Hellman key exchange protocol is one-way functions, that is, functions that are simple to calculate in one direction, but require relatively large resources to calculate in the opposite direction. The secret key created based on the key exchange protocol can later be used to encrypt data.

Another interesting method for generating a shared secret key is based on matrix algebra. In general, the essence of this method is based on building a public-key exchange process based on matrices whose inverse cannot be calculated. It is known that the product of two matrices can be calculated only if the number of columns in the first matrix is equal to the number of rows in the second matrix. In the special case, if both matrices are square matrices of the same size, then their product can always be found.

When matrices A and X satisfying these conditions are multiplied together, the matrix C of the corresponding size is obtained:

$$A \cdot X = C \quad (1)$$

If C and A are known, the matrix X can be calculated based on the following expression:

$$X = C \cdot A^{-1} \quad (2)$$

Here, the matrix A^{-1} is the inverse matrix of the matrix A. If we denote the determinant of the matrix A by *detA*, and the minor by A_{nm} (i,j=1,2,...,n), the following expression is used to find the inverse matrix:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix} \quad (3)$$

As can be seen, if *det*A=0, it is necessary to perform the division by zero operation, which leads to uncertainty, that is, it is impossible to calculate the inverse of the matrix. In other words, although it is always possible to find the product of two matrices when one of the factors and the product are given, in some cases it is impossible to calculate the other factor. This is because if the known factor is a matrix that does not have an inverse, then for the other factor there are infinitely many solutions that satisfy equation (1). It is this property of matrices that allows us to create a one-way function with their help [3-4]. In the studies conducted by the authors in the literature [2], a key exchange protocol based on non-invertible matrices was proposed. In this paper, the issue of assessing the security and efficiency of this protocol is considered.

The essence of the key exchange protocol based on non-invertible matrices given in [2] is as follows:

1) the parties (Parties 1 and 2) who want to exchange information choose a common square matrix C with determinant equal to 0;

2) The first party chooses an nxn matrix A and a coefficient q, and the second party chooses a matrix B and a coefficient p. The first party calculates the product $S_{A1} = A \cdot C^q$ and sends it to the second party, and the second party calculates the product $S_{B1} = C^p \cdot B$ in a similar manner to the first party.

3) The first party chooses an nxn matrix M and multiplies the product $M \cdot C^q$ by the matrix S_{B1} obtained from the second party from the left, and sends the resulting matrix $S_{A2} = M \cdot C^q \cdot C^p \cdot B$ to the second party. The second party selects an nxn matrix N and sends the matrix $S_{B2} = A \cdot C^q \cdot C^p \cdot N$ obtained by multiplying the product $C^p \cdot N$ by the matrix S_{A1} from the right to the first party.

4) The first party multiplies the product $M \cdot A^{-1}$ from the second party to the matrix S_{B2} from the left, and the second party multiplies the product $B^{-1} \cdot N$ from the first party to the matrix S_{A2} from the right. Both parties get the same result: $S = M \cdot C^q \cdot C^p \cdot N$. The resulting matrix is used as a secret key that the parties jointly form.

2. LINEAR ALGEBRA-BASED SECURITY ANALYSIS

Since the intermediate keys, S_{A1} and S_{A2}, are obtained from the product of the matrix C by another matrix, they must be non-invertible matrices (the product of non-invertible and inverse matrices is equal to the non-invertible matrix). Therefore, it is not possible to calculate the matrix

$M \cdot C^q$ from the equation $S_{A2} = M \cdot C^q \cdot S_{B1}$. Because the number of matrices $M \cdot C^q$ satisfying this equation is greater than one. If the matrices M and N are not used in the protocol, the equation $S_{A2} = M \cdot C^q \cdot C^p \cdot B$ takes the form $S_{A2} = C^q \cdot S_{B1}$. Then the number q can be calculated by checking all possible values until the equation $S_{A2} = C^q \cdot S_{B1}$ is satisfied. The number p can be calculated in the same way. However, when the matrix M is included in the equation, it is necessary to find not only the number q, but also the matrix M. This increases both the number of cases to be checked and the number of solutions to the equation $S_{A2} = M \cdot C^q \cdot C^p \cdot B$. Thus, it becomes extremely difficult to verify the correctness of the solution found. By the same logic, the importance of including the matrix N can be clearly shown.

3. KEY FİELD ANALYSİS

The brute force resistance of the proposed method directly depends on the number of non-inverse matrices that can be used. To calculate this number, let us assume that the elements of the matrix Z with any inverse take values from 0 to 999. Then the first column of the matrix Z can take $(1000^n – 1)$ different values (not every element in the first column can be equal to 0 at the same time). The second column must be outside the one-dimensional space formed by the first column. That is, if we denote the first column by a_1, then the second column must be different from a_1, $2 * a_1$, $3 * a_1$, ... , $1000 * a_1$. Therefore, a_2 can take $(1000^n – 1000)$ values. The third column must be outside the space formed by the vectors a_1 and a_2. That is, it can take $(1000^n – 1000^2)$ values. Continuing in a similar way, we see that the value of the vector a^i is $(1000^n – 1000^i)$. Taking these into account, we can see that the number of possible values of the matrix Z is $(1000^n – 1) * (1000^n – 1000) * (1000^n – 1000^2) * ... * (1000^n – 1000^{n-1})$. If the elements of the matrix Z can take the value "m", then the number of such matrices is $(m^n – 1) * (m^n – m) * (m^n – m^2) * ... * (m^n – m^{n-1})$. The number of non-invertible matrices is equal to the difference between the number of all matrices and the number of invertible matrices. Given that the number of all matrices is m^{n*n}, the number of non-invertible matrices is equal to $m^{n*n} - (m^n – 1) * (m^n – m) * (m^n – m^2) * ... * (m^n – m^{n-1})$. Table 1 shows the number of non-invertible matrices depending on the size of the matrix and the largest value of its elements. The results prove that the algorithm is quite strong against brute force cracking. For example, for matrices of size 4x4 and 5x5 with 4-digit elements, the numbers 10^{60} and 10^{96} were obtained, which correspond to 200 and 319-bit numbers when represented in binary code. As can be seen, the key area of the algorithm is quite large, and if necessary, this area can be further increased by changing the size of the matrix and the range of values that its elements take.

Table 1. The number of non-invertible matrices based on the matrix size and the maximum value of the element

Matrix size	Maximum value of the elements of the matrix			
	99	999	9999	99999
2	$1.009 * 10^6$	10^9	10^{12}	10^{15}
3	$1.009 * 10^{16}$	10^{24}	10^{32}	10^{40}
4	$1.009 * 10^{30}$	10^{45}	10^{60}	10^{75}
5	$1.009 * 10^{48}$	10^{72}	10^{96}	10^{120}

4. EVALUATION OF THE RESOURCES USED

One of the main parameters evaluated is the determination of the time spent on key generation. As can be seen from the algorithm, a very large part of the resource is spent on calculating the product of matrices. The size of the matrices and the maximum value that the elements can take significantly affect the time spent on calculating the common key. Naturally, the characteristics of the computer on which the program is executed also play a major role here. The results obtained by implementing the program designed for the algorithm under consideration on a computer with the parameters *CPU: Intel® Core™ i5-8265U CPU @ 1.60GHz, RAM: 8 GB* are given in Table 2. As can be seen, the results are quite high for such a process and allow calculating a separate key for each information exchange session.

Table 2. Time spent on key formation (*in seconds*)

Matrix size	Maximum value of the elements of the matrix				
	100	1000	10000	100000	1000000
3	0.002627849	0.002312302	0.002815985	0.002208733	0.002175283
4	0.006254410	0.006436371	0.006414818	0.006233954	0.006390404
5	0.056668591	0.054777193	0.059532523	0.058203577	0.057142686
6	0.114728498	0.118162989	0.119587588	0.120708131	0.124338245
7	0.230506992	0.233087420	0.230650210	0.231170415	0.254934334

5. CONCLUSION

The article evaluates the security of the protocol for generating a shared secret key based on non-invertible matrices. The evaluation is based on linear algebra and key space analyses. In the presented article, the time spent on implementing the protocol based on the application of non-invertible matrices is also evaluated. The analyses have shown that the calculation of the shared secret key by the considered method has high resistance to linear algebra and brute force attacks. The proposed approach, in addition to providing a high level of security for key exchange, has practical efficiency. In future work, it is planned to investigate and optimize wider application areas of the protocol.

REFERENCES

1. Gasimov V.A. Fundamentals of information security. Textbook, Baku, 2009, 340 p.
2. Gasimov V., Mammadzada N. and Mammadov J., "New Key Exchange Protocol Based on Matrix Algebras", *2023 5th International Conference on Problems of Cybernetics and Informatics (PCI)*, Baku, Azerbaijan, 2023, pp. 1-3, doi: 10.1109/PCI60110.2023.10326004.
3. Rahman, Nael and Shpilrain, Vladimir. "MAKE: A matrix action key exchange" *Journal of Mathematical Cryptology*, vol. 16, no. 1, 2022, pp. 64-72. https://doi.org/10.1515/jmc-2020-0053
4. Daniel Brown, Neal Koblitz & Jason LeGrow. (2021) Cryptanalysis of `MAKE', Cryptology ePrint Archive, Paper 2021/465, https://eprint.iacr.org/2021/465

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INTERACTION TECHNOLOGIES AND APPLICATIONS
(INTERACTION TECHNOLOGY IS EASILY UNDERSTOOD BY PEOPLE)

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ABSTRACT

The thesis discusses interactive technologies for creating easy-to-understand programs by minimizing latency. What interactive technology is and where it is used have been investigated. Which technologies can be interactive, how can we create interactive programs with basic elements, the design of interactive applications, its negative and positive aspects, and such issues are analyzed. Its primary benefit in the management field is explained, and its applications to increase efficacy in various sectors are also discussed. In this thesis, we can see the importance of the interactive approach and its positive impact on other areas. At the same time, we will witness that future life will be mostly interactive, and people will reach their goals more easily thanks to applications. It is emphasized that the main starting point for the interaction of developed technologies is good design, and what are the necessary conditions for this design.

Introduction

Humans use more sophisticated interfaces to communicate with technology, progressing from standard keyboards to touchscreens, voice commands, and beyond. Even these engagement patterns are being replaced by fresh, easier, and more organic ways to communicate. Images and video streams, for instance, may be utilized to track assets, confirm personal identities, and comprehend the context of surrounding locations. Enhanced speech capabilities enable natural, nuanced discussions with complicated systems. Additionally, AI-based systems may respond to nonverbal user orders by interpreting human gestures, head movements, and gazes. Modern human-centered design methodologies are combined with cutting-edge technology like computer vision, conversational speech, audio analytics, and enhanced augmented reality and virtual reality to create intelligent interfaces. Together, these methods and tools are revolutionizing how we interact with technology, information, and one another.

What is interactive technology and app?

Here is a simple explanation of how Interactive Technology works as letters and individual words. They serve as the foundation for the tale you want to tell and the feelings you want to elicit.

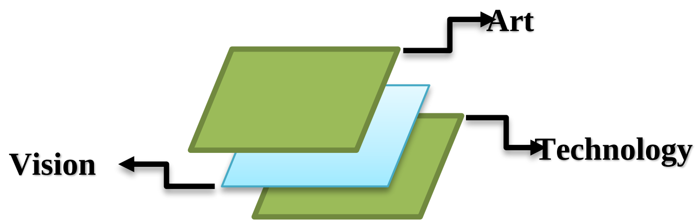
There are several ways to characterize interactive technology. For my purposes, I'll define it as follows: Technology designed and built for real-time applications, with an emphasis on human/user input and environmental sensing. Interactive technology enables a two-way flow of information between the user and the technology via an interface; the user typically sends a request for data or action to the technology, with the technology returning the requested data or the outcome of the action to the user.[1][10]

An interactive application is one in which you enter information and receive instant results. As a result, the program interacts with you and what you enter. The program should reply to your input promptly.

What makes a great interactive application

Interactive applications are essentially any application where you input information and you gain immediate output. Thus, the application interacts with you and what you input. The application should respond to your input immediately.

1. Art



An interactive app's major component is art. Because a projection is just how we perceive and experience something, how we feel it. The artist who discovers the perfect pattern that fits the people's ideas becomes renowned. Let's look at a real-world case. When we meet new individuals, we are drawn to their outward looks. Next, we take a look inside. This is frequently the case. As a result, art assists people in seeing the consequences in the most attractive way possible.[1]

2. Vision

We all see the world through the lenses of our senses. We hear, feel, and do a variety of other things. Vision is perhaps the most crucial mode of perception for humans. Vision connects our brain to the outside world, making it much simpler to engage with it. Blind individuals see the world through their other senses, which are significantly more sophisticated.

3. Technology

We now have a variety of tools to assist us to comprehend each other and the world around us through its projection. We have books, radio, television, computers, and smartphones. We all adore cell phones. We can carry the entire world's information in our pockets.

We began with a grayscale flat snake game and have moved to 3D colorful games with HDR and a wealth of other inventive applications with projection details. We can now see a courier driving a vehicle on a 3D map and accurately anticipate the delivery time. The primary issue, here is 3D. Even on a flat screen, humans may perceive things that are remarkably close to their original forms and proportions. Remember the growth of Google Maps? Initially, we see flat maps of certain towns, then countries, then the entire planet, and ultimately, satellite photos materialize in front of us. We are seeing more and more 3D on flat maps. But why did they decide to add 3D when we already have satellite images? The solution is straightforward: we require a better representation of the real environment. Images are not interactive; they cannot be rotated and always display the same projection. With 3D maps, you can view anything from many perspectives, giving you a comprehensive grasp of many real-life elements.[1]

Use of interactive technology

The applications of interactive technology are numerous and diverse. They are used in a variety of settings, including education, training, marketing, and data collection. Smartphones are the best example. The smartphone is a powerhouse of interactive technologies, and it is so widely used that people of all ages, nationalities, world views, and so on have used one. Regardless of brand or model, many of them use the same fundamental technologies. They have touch screens with gesture controls. These are excellent examples of interactive technologies in use in our daily lives.

1. Education and training

Interactive technology in classrooms is a type of educational technology (EdTech) that allows students to not only view information but also interact with the content in their lesson. With new educational technology tools being developed on a daily basis, it is becoming easier than ever to create interactive classrooms. It could be a while before we see interactive whiteboards in every classroom or individual tablets on every student's desk. However, an increasing number of schools, universities, and colleges are utilizing interactive technology to transform the way their students learn.

2. Marketing

Over traditional methods of gathering consumer data, interactive technology has two distinct advantages. For starters, it allows information to be gathered in real-time, allowing for faster responses to customers than traditional media. The more orders a consumer places on Amazon.com, for example, the more information about that consumer's reading preferences is gathered. There are numerous benefits to employing interactive marketing, especially given that customers expect businesses to surpass their expectations. Organizations that use interactive marketing have a better chance of addressing the needs of their customers because they have already expressed an interest in the product and marketers can respond to their actions. Because it is based on customer behaviors and wishes, interactive marketing decreases risk and boosts sales. Personalization in interactive marketing leads to increased conversions and higher income. Ultimately, interactive marketing can increase sales, improve consumer satisfaction, reduce marketing expenses, and pave the way for automated marketing

3. Information gathering

Traditional approaches benefit from interactive documents. Surveys that try to evaluate satisfaction with new product expectations and answers can be more effective when conducted using interactive multimedia. In the above scenario, Amazon.com would have more trustworthy information about a consumer's choices than any paper survey it could ask the public to complete. These surveys may collect more information since they are more fascinating than their paper counterparts.

Interactive design of technologies

Designing interactive products to support the way people communicate and interact in their every day and working lives. Interaction design relies on an understanding of the capabilities and desires of people and on the kinds of technology available to interaction designers, as well as the knowledge of how to identify the requirements and evolve them into a suitable design.

Designing visible interactive products requires considering who is going to be using them, how they are going to be used, and where they are going to be used. Another key concern is understanding the kind of activities people are doing when interacting with the products. The appropriateness of different kinds of interfaces and arrangements of input and output devices depends on what kinds of activities need to be supported. A key question for interaction design is: how do you optimize the users' interactions with a system, environment, or product so that they support and extend that users' activities in effective, useful, and usable ways? One could use institutions and hope for the best. Alternatively, one can be more principled in deciding which choice to make by basing them on an understanding of their users.[8] This involves:

- Taking into account what people are good and bad at.
- Considering what might help people with the way they currently do things.
- Thinking through what might provide quality user experiences.
- Listening to what people want and getting them involved in the design.
- Using 'tired and tested' user-based techniques during the design process. [6]

The Rise and Impact of Interactive Technologies

1. Enhanced Communication:

Interactive technologies have drastically transformed communication, making it more instantaneous and immersive. Social media platforms, video conferencing tools, and messaging apps have connected people across the globe, transcending physical boundaries. The ability to share information, images, and videos in real-time has facilitated the exchange of ideas, fostered collaboration, and enriched personal connections. Moreover, interactive technologies have given rise to virtual communities, enabling individuals with shared interests to connect and interact, regardless of their geographic location.

2. Revolutionizing Entertainment:

The entertainment industry has been profoundly impacted by interactive technologies. Video games, for instance, have evolved from simple pixelated graphics to immersive virtual worlds, powered by advanced graphics, motion sensors, and haptic feedback. Players can actively engage with the virtual environment, controlling characters and influencing narratives, creating a more personalized and immersive experience.

3. Transforming Education:

Interactive technologies have revolutionized education, enhancing the learning experience and expanding access to knowledge. Interactive whiteboards, digital textbooks, and online learning platforms have made education more engaging and interactive. Students can now participate actively in their learning process, accessing multimedia resources, simulations, and interactive exercises. Virtual reality and augmented reality have taken this transformation even further, enabling students to explore historical sites, conduct virtual experiments, and experience immersive simulations, making learning more memorable and impactful.[9]

4. Reshaping Business:

Interactive technologies have disrupted traditional business models, opening up new opportunities for innovation and growth. E-commerce platforms have revolutionized the way people shop, providing personalized recommendations, immersive product experiences, and convenient payment options. Augmented reality applications have allowed customers to try on virtual clothes, visualize furniture in their homes, and preview products before making purchasing decisions.

5. Advancements in Healthcare:

Interactive technologies have played a crucial role in healthcare, improving diagnostics, treatment, and patient care. Virtual reality and augmented reality have been employed to simulate surgeries, train medical professionals, and assist in rehabilitation. Telemedicine has become increasingly popular, allowing patients to consult with doctors remotely, reducing barriers to accessing healthcare services. Wearable devices and mobile apps have empowered individuals to monitor their health and receive personalized feedback, promoting preventive care and well-being.

The Rise and Impact of Interactive Apps

Interactive apps have experienced a significant rise in popularity, transforming the way we engage with technology and consume digital content. These apps provide dynamic and engaging experiences, allowing users to actively participate and manipulate the content, creating a more personalized and immersive environment. This essay explores the rise and impact of interactive apps across various industries, including education, entertainment, productivity, and social networking.

1. Education:

Interactive apps have revolutionized the education sector, making learning more accessible and engaging. These apps offer interactive content, simulations, quizzes, and puzzles that cater to different learning styles and encourage active participation. Students can access educational material anytime, anywhere, and at their own pace. Gamification elements, such as badges and rewards, motivate learners and enhance their learning experience. Interactive apps have also enabled distance learning and remote education, providing access to quality education beyond traditional classrooms.[4]

2. Entertainment:

Interactive apps have had a profound impact on the entertainment industry, transforming the way we consume and interact with media. Gaming apps, in particular, have seen tremendous growth, offering immersive and interactive experiences across various genres.

3. Productivity:

Interactive apps have significantly impacted productivity by providing tools and features that enhance efficiency and organization. Productivity apps, such as task managers, note-taking apps, and project management tools, offer interactive interfaces that allow users to easily manage and prioritize tasks. These apps often provide collaboration features, enabling teams to work together seamlessly.

4. Health and Well-being:

Interactive apps have made a significant impact on health and well-being by promoting active lifestyles, mental wellness, and personalized health management. Fitness apps offer interactive workout routines, tracking progress, and providing real-time feedback. Meditation and mindfulness apps offer interactive features that guide users through relaxation techniques and stress management exercises.

Disadvantages of Interactivity

While interactive technologies have brought numerous benefits and advancements, it is essential to recognize that there can be negative sides to their use. Here are some potential drawbacks and concerns associated with interactive technologies:

1. Social Isolation:

As people spend more time engaging with interactive technologies, there is a risk of increased social isolation. Excessive use of interactive apps and devices may lead to decreased face-to-face interactions, causing individuals to feel disconnected from real-life relationships. This isolation can have detrimental effects on mental health and interpersonal skills.

2. Physical Health Issues:

Extended use of interactive technologies, particularly gaming apps and virtual reality systems, can contribute to sedentary behavior and a lack of physical activity. Prolonged screen time and poor posture can lead to musculoskeletal problems, obesity, and other health issues. Additionally, excessive exposure to screen-based interactive apps may disrupt sleep patterns and contribute to sleep disorders.

3. Privacy and Security Concerns:

Interactive apps often require access to personal information and data. There is a risk of this data being misused or falling into the wrong hands, leading to privacy breaches, identity theft, and other security concerns. Additionally, the collection and storage of personal data by interactive apps raise ethical questions regarding consent, transparency, and the use of personal information for targeted advertising.

4. Inequality and Access:

Not everyone has equal access to interactive technologies due to factors such as economic disparities and limited digital infrastructure. This can create a digital divide, exacerbating existing inequalities. Those without access to interactive apps may face disadvantages in terms of education, employment opportunities, and social inclusion.[9]

Conclusion

Interactive technologies have become an integral part of our lives, revolutionizing communication, entertainment, education, business, and healthcare. These technologies have enhanced our ability to connect, learn, and engage with the world around us, opening up new possibilities and transforming traditional practices. As interactive technologies continue to advance, it is crucial to ensure their responsible and ethical use, addressing concerns related to privacy, security, and social impact. By harnessing the potential of interactive technologies wisely, we can unlock a future that is more connected, immersive, and enriching for individuals and societies worldwide.

Interactive apps have experienced a remarkable rise in popularity, transforming various industries and impacting our daily lives. From education and entertainment to productivity and social networking, these apps have revolutionized the way we engage with technology and consume digital content. With their dynamic and immersive experiences, interactive apps have enhanced learning, entertainment, productivity, and well-being. As technology continues to advance, interactive apps will continue to shape the future of digital experiences, providing innovative and engaging solutions for users around the world.

Finally, interactive apps engage and capture the audience, give the proper message to consumers, and extend the life of the product. We can conclude that interactive technology manages to be at the center of our lives in a variety of ways, including education, marketing, and, most notably, cell phones, and has had a significant positive impact on my life. It is dependent on how we employ it.

REFERENCES

[1] Slava Podmurniy; "What is Interactive App and Why It Matters for Your Business", https://www.visartech.com/blog/what-is-interactive-app-and-how-business-benefits/

[2] Interactive Application - an overview | ScienceDirect Topics

[3] https://www.simplilearn.com/iot-devices-article#what_is_iot

[4] Elbruz, Interactive Technology, December 14, 2015, https://interactiveimmersive.io/blog/beginner/02-interactive-technology/

[5] Emily Esposito, 10 amazing examples of interaction design, Aug 20, 2018, https://www.invisionapp.com/inside-design/amazing-interaction-design/

[6] Alan Smith, A brief introduction to interaction design, https://usabilitygeek.com/introduction-interaction-design/

[7] https://www.encyclopedia.com/finance/finance-and-accounting-magazines/interactive-technology

[8] "Interactive Design". John Wiley and Sons, Ltd (Book)

[9] https://elearningindustry.com/evolution-of-education-how-interactive-technologies-reshaped-learning

[10] Natalia De la Peña Frade, Please, can someone tell me what interactivity is? | Genially Blog

[11] What Are Interactive Technologies? | 2023 (michiganstopsmartmeters.com)

2024. Volume 8, Number 2 Pages 112-119

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EXISTENCE OF A SOLUTION TO A MIXED PROBLEM FOR A PARABOLIC EQUATION IN THE SENSE OF SHILOV

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ABSTRACT

Mixed problem for the fourth order ordinary differential equation with general boundary conditions is considered in present paper. Soluion of the problem is found by the residue method. According to the scheame of this method the mixed problem is divided by two auxiliary- specrtal and Cauchy problems. After researching these two problems, solution of the considering mixed problem is found by residue series. It is shown , that solution of considering mixed problem surround not only parabolic equations in the sense of Shilov, but also wider classes of equations.

Introduction

The fourth-order harmonic Schrödinger equation is of great importance in the study of wave processes. These equations can be transformed into parabolic type equations in the sense of Petrovsky [4,5,11,12]. Additionally, there are more general forms of parabolic equations beyond those defined by Petrovsky, such as parabolic equations in the sense of Shilov [7,8,9,10].

İn equations of this type, the inclusion of the potential function can alter the nature of the equation. In other words, wave processes can be analyzed within the framework of parabolic equations in the sense of Shilov, and the fourth-order equation we consider falls into this class. [1]

For instance, examining the heat transfer process in rods of the same length but with different heat transfer coefficients can be represented by fourth-order partial differential equations [5,6].

The fourth-order parabolic equations in the sense of Shilov.

Consider the following problem:

$$\frac{\partial u(x,t)}{\partial t} = ip\frac{\partial^4 u(x,t)}{\partial x^4} + e^{ix}\frac{\partial^2 u(x,t)}{\partial x^2}, \; 0 < x < 1, \; t > 0 \quad (1)$$

$$u(x,0) = \varphi(x), \quad (2)$$

$$L_m(u) \equiv \sum_{k=1}^{4}\left(\alpha_{mk}\frac{\partial^{k-1}u(0,t)}{\partial^{k-1}x} + \beta_{mk}\frac{\partial^{k-1}u(1,t)}{\partial^{k-1}x} \right) = 0, \quad m = \overline{1,4} \tag{3}$$

where α_{mk}, β_{mk} ($m = \overline{1,4}$, $k = \overline{1,4}$) are complex numbers, $p > 0$ *real number*, $\varphi(x)$ is complex valued functions.

After application integral transformation

$$y(x,\lambda) = \int_0^\infty u(x,t)e^{-\lambda^4 t}dt$$

to the problem (1)-(3), we'll get following spectral problem:

$$ipy^{IV} + q(x)y'' - \lambda^4 y = -\phi(x), \quad 0 < x < 1 \tag{4}$$

$$L_m(y) \equiv \sum_{k=1}^{4}\left(\alpha_{mk}y^{(k-1)}(0,\lambda) + \beta_{mk}y^{(k-1)}(1,\lambda)\right) = 0, \quad m = \overline{1,4}\ . \tag{5}$$

Roots of the characteristic equation in the sense of Birkhof corresponding to the equation (4) are found as follows [2]:

$$\theta_1 = p^{-\frac{1}{4}} \cdot e^{-\frac{\pi}{8}i}, \ \theta_2 = i\,\theta, \ \theta_3 = -\theta_1, \ \theta_4 = -i\,\theta_1\ .$$

To find asymptotic of fundamental solutions of the equation (4) let's devide a complex plane λ into eight sectors by the following way [7]:

$$S_k = \left\{\lambda : -\lambda_1\, tg\frac{\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{\pi}{8}\right\},\ k = 1,2,$$

$$S_k = \left\{\lambda : \lambda_1\, tg\frac{\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{3\pi}{8}\right\},\ k = 3,4,$$

$$S_k = \left\{\lambda : \lambda_1\, tg\frac{3\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{5\pi}{8}\right\},\ k = 5,6,$$

$$S_k = \left\{\lambda : \lambda_1\, tg\frac{5\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{7\pi}{8}\right\},\ k = 7,8.$$

At the each sectors $S_k\ (k = \overline{1,8})$ at large values of $|\lambda|$ the asymptotics of fundamental solution of the equation (4) have the following representation [6, 8]:

$$\frac{d^s y_n(x,\lambda)}{dx^s} = (\lambda\theta_n)^s\left[1 + \frac{1}{4i\theta_n\lambda}(e^{ix} - 1) + O\left(\frac{1}{\lambda^2}\right)\right]e^{\lambda\theta_n x},$$
$$|\lambda| \to +\infty,\ \lambda \in S_m\ (m = \overline{1,8}),\ n = \overline{1,4},\ s = \overline{0,3}. \tag{6}$$

Green function of the spectral problem (4), (5) has the form [4] :

$$G(x,\xi,\lambda) = \frac{\Delta(x,\xi,\lambda)}{\Delta(\lambda)}\ ;\ \lambda \in S_m,\ m = \overline{1,8}. \tag{7}$$

$\Delta(\lambda)$ is called a characteristic determinant and is found as follows

$$\Delta(\lambda)=\begin{vmatrix} L_1(y_1) & L_1(y_2) & L_1(y_3) & L_1(y_4) \\ L_2(y_1) & L_2(y_2) & L_2(y_3) & L_2(y_4) \\ L_3(y_1) & L_3(y_2) & L_3(y_3) & L_3(y_4) \\ L_4(y_1) & L_4(y_2) & L_4(y_3) & L_4(y_4) \end{vmatrix}$$

(8) and auxiliary determinant $\Delta(x,\xi,\lambda)$ is found as follows

$$\Delta(x,\xi,\lambda)=\begin{vmatrix} g(x,\xi,\lambda) & y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & y_4(x,\lambda) \\ L_1(g)_x & L_1(y_1) & L_1(y_2) & L_1(y_3) & L_1(y_4) \\ L_2(g)_x & L_2(y_1) & L_2(y_2) & L_2(y_3) & L_2(y_4) \\ L_3(g)_x & L_3(y_1) & L_3(y_2) & L_3(y_3) & L_3(y_4) \\ L_4(g)_x & L_4(y_1) & L_4(y_2) & L_4(y_3) & L_4(y_4) \end{vmatrix},$$

(9)

where Cauchy function $g(x,\xi,\lambda)$ is found as follows [5]

$$g(x,\xi,\lambda)=\pm\frac{1}{2}\sum_{k=1}^{4} z_k(\xi,\lambda)y_k(x,\lambda)$$

"+" if $0\le\xi\le x\le 1$, "-" if $0\le x\le\xi\le 1$,

here $z_k(\xi,\lambda)=\dfrac{V_{4k}(\xi,\lambda)}{V(\xi,\lambda)}$, $k=\overline{1,4}$,

$V_{4k}(\xi,\lambda)$ is an algebraic complement of the fourth row element of Vronskian $V(\xi,\lambda)$.

To find the asymptotic of eigenvalues of spectral problem (4), (5) let's introduce the following notations :

$$L(\gamma^1_{k_1}\gamma^2_{k_2}\gamma^3_{k_3}\gamma^4_{k_4})=\begin{vmatrix} \gamma^1_{1k_1} & \gamma^2_{1k_2} & \gamma^3_{1k_3} & \gamma^4_{1k_4} \\ \gamma^1_{2k_1} & \gamma^2_{2k_2} & \gamma^3_{2k_3} & \gamma^4_{2k_4} \\ \gamma^1_{3k_1} & \gamma^2_{3k_2} & \gamma^3_{3k_3} & \gamma^4_{3k_4} \\ \gamma^1_{4k_1} & \gamma^2_{4k_2} & \gamma^3_{4k_3} & \gamma^4_{4k_4} \end{vmatrix},$$

$$A_0=2L(\alpha_2\alpha_3\beta_2\beta_3),$$
$$B_0=2\big(L(\alpha_2\alpha_3\beta_1\beta_3)-L(\alpha_1\alpha_3\beta_2\beta_3)\big),$$
$$C_0=2\big(L(\alpha_1\alpha_2\alpha_3\beta_3)+L(\alpha_3\beta_1\beta_2\beta_3)\big),$$
$$g_k(x)=\frac{1}{4\theta_k}(e^{ix}-1),\ k=\overline{1,4}.$$

Now to find asymptotic of eigenvalues of spectral problem (4), (5) consider the following theorem:

Theorem1. Suppose,that α_{mk}, β_{mk} $(m=\overline{1,4}\ ;\ k=\overline{1,3})$ are complex numbers. Then the zeros of the characteristic determinant $\Delta(\lambda)$ are countable set, single limit points of which is $\lambda=\infty$ and the following formulas for the asymptotic zeros are true:

$$\lambda_n^4 = \left(4n^4+8n^3+6n^2\right)\pi^4 i - \pi^2 n^2\left(\sin 1 - i(\cos 1 - 1) + 4i\frac{B_0 \pm C_0}{A_0}\right) + O(n) \quad n \to \pm\infty \tag{10}$$

Proof

Based on the property of determiant, the $\Delta(\lambda)$, found by formula (8) can be rewritten as follows:

$$\Delta(\lambda) = D_{12}(\lambda)e^{(\theta_1+\theta_2)\lambda} + D_{14}(\lambda)e^{(\theta_1+\theta_4)\lambda} + D_{23}(\lambda)e^{(\theta_2+\theta_3)\lambda} + D_{34}(\lambda)e^{(\theta_3+\theta_4)\lambda} + \dots\dots\dots + \sum_{k=1}^{4} D_k(\lambda)e^{\theta_k(\lambda)} + D_0(\lambda). \tag{11}$$

To find the main part of determinant $\Delta(\lambda)$ let's use the traditional method, that is equate the real part of exponents in pairs and selecting the straight lines or semi-straights, we'll get [1,3]:

$$\lambda_2 = \lambda_1 tg\left(\frac{\pi}{8} + \frac{\pi}{4}(k-1)\right),\ k=\overline{1,8},\quad |\lambda_1| > R.$$

Choose those of the semi-strips, constructed from the above-mentioned semi-straights, where the main part of $\Delta(\lambda)$ has an infinite number of zeroes. Let's denote as $\Pi_k(\lambda)$ and $\Delta_k(\lambda)$ $(k=\overline{1,4})$ these kinds of semi-strips and defined there the main parts of the determinant $\Delta(\lambda)$, correspondingly. Thus, the semi-strips $\Pi_k(\lambda)$ and $\Delta_k(\lambda)$ -the main part of $\Delta(\lambda)$ are defined as follows:

Consider the main part of $\Delta(\lambda)$ in the first quarter [4]:

$$\Pi_1(\lambda) = \left\{\lambda = \lambda_1 + i\lambda_2 : -\delta < \lambda_2 - \lambda_1 tg\frac{\pi}{8} < \delta,\ \lambda_1 > R\right\},$$

$$\Delta_1(\lambda) = \theta_1^9 \lambda^{10}\left[\left(i\theta_1 A_0\left(1+(1-i)g_1(1)\frac{1}{\lambda}\right) + (1+i)B_0\frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_2)\lambda} + \right.$$
$$\left. + \left(i\theta_1 A_0\left(1+(1+i)g_1(1)\frac{1}{\lambda}\right) + (-1+i)B_0\frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_4)\lambda} + \left(2iC_0 + O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_1\lambda}\right].$$

In the second quarter the main part has the form:

$$\Pi_2(\lambda) = \left\{\lambda = \lambda_1 + i\lambda_2 : -\delta < \lambda_2 - \lambda_1 tg\frac{5\pi}{8} < \delta,\ \lambda_1 > R\right\}$$

$$\Delta_2(\lambda) = \theta_1^9 \lambda^{10}\left[\left(i\theta_1 A_0\left(1+(1-i)g_1(1)\frac{1}{\lambda}\right) + (1+i)B_0\frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_2)\lambda} + \right.$$
$$\left. + \left(i\theta_1 A_0\left(1+(-1-i)g_1(1)\frac{1}{\lambda}\right) + (1-i)B_0\frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_2+\theta_3)\lambda} + \left(2iC_0\frac{1}{\lambda} + O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_2\lambda}\right].$$

The main part in the third quarter has the form

$$\Pi_3(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{\pi}{8}<\delta,\ \ \lambda_1<-R\right\}$$

$$\Delta_3(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1 A_0\left(1+(-1-i)g_1(1)\frac{1}{\lambda}\right)+(1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_2+\theta_3)\lambda}+\right.$$

$$\left.+\left(i\theta_1 A_0\left(1+(-1+i)g_1(1)\frac{1}{\lambda}\right)+(-1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_3+\theta_4)\lambda}+\left(-2iC_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_3\lambda}\right]$$

$$\Pi_4(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{5\pi}{8}<\delta,\ \ \lambda_1<-R\right\}$$

$$\Delta_4(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1 A_0\left(1+(1+i)g_1(1)\frac{1}{\lambda}\right)+(-1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_4)\lambda}+\right.$$

$$\left.+\left(i\theta_1 A_0\left(1+(-1+i)g_1(1)\frac{1}{\lambda}\right)+(-1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_3+\theta_4)\lambda}+\left(-2C_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_4\lambda}\right],$$

here $\delta>0$ and R is sufficiently large number.

Firstly, solve equation $\Delta_1(\lambda)=0$. For that introduce following notations:

$$\Delta_{11}(\lambda)=\left(i\theta_1 A_0 e^{\theta_2\lambda}+i\theta_1 A_0 e^{\theta_4\lambda}\right)\theta_1^9\lambda^{10}e^{\theta_1\lambda},$$

$$\Delta_{10}(\lambda)=\Delta_1(\lambda)-\Delta_{11}(\lambda)=\theta_1^9\lambda^{10}\left(i\theta_1 A_0(1-i)g_1(1)+(1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda}\right)\right)e^{(\theta_1+\theta_2)\lambda}+$$

$$\left.+\left(i\theta_1 A_0(1+i)g_1(1)+(-1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{(\theta_1+\theta_4)\lambda}+\left(2iC_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_1\lambda}\right].$$

$\mu_{1n}=\left(\frac{\pi}{2}+\pi n\right)\frac{1}{\theta_1}$, $n\to+\infty$. After solution of the equation $\Delta_{11}(\lambda)=0$ we will get:

To find roots of the equation $\Delta_1(\lambda)=0$ consider a following formulas [9]

$$\lambda_{1n}^r=\mu_{1n}^r+r\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\underset{\lambda=\mu_{1n}}{res}\left[\lambda^{r-1}\left(\frac{\Delta_{10}(\lambda)}{\Delta_{11}(\lambda)}\right)^m\right].$$

In case of $r=4$ and $m=1$ we will get

$$\lambda_{1n}^4=\mu_{1n}^4-4\underset{\lambda=\mu_{1n}}{res}\lambda^3\frac{\Delta_{10}(\lambda)}{\Delta_{11}(\lambda)}=\mu_{1n}^4-4\underset{\lambda=\mu_{1n}}{res}\lambda^2\times\left[\left(i\theta_1 A_0(1-i)g_1(1)+(1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_2\lambda}+\right.$$

$$\left.+\left(i\theta_1 A_0(1+i)g_1(1)+(-1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_4\lambda}+\left(2iC_0+O\left(\frac{1}{\lambda}\right)\right)\right]/\left(i\theta_1 A_0 e^{\theta_2\lambda}+i\theta_1 A_0 e^{\theta_4\lambda}\right).$$

It is easy to check, that μ_{1n} are simple poles of the function $\Delta_{11}(\lambda)$. According to that, we'll get

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2 \frac{\theta_1 A_0 g_1(1)\left((1-i)e^{\theta_2\mu_{1n}} + (-1+i)e^{\theta_4\mu_{1n}}\right)e^{\theta_2\lambda} + B_0\left((1+i)e^{\theta_2\mu_{1n}} + (-1+i)e^{\theta_4\mu_{1n}}\right) + 2iC_0}{i\theta_1\theta_2 A_0\left(e^{\theta_2\mu_{1n}} - e^{\theta_4\mu_{1n}}\right)},$$

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2\left[\frac{g_1(1)}{i\theta_2} + \frac{B_0}{i\theta_1\theta_2 A_0} - \frac{2C_0}{\theta_1\theta_2 A_0\left(e^{\theta_2\mu_{1n}} - e^{-\theta_4\mu_{1n}}\right)}\right],$$

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2\left[\frac{g_1(1)}{i\theta_2} + \frac{B_0}{i\theta_1\theta_2 A_0} - \frac{C_0}{\theta_1\theta_2 A_0 \sin\left(\frac{\pi}{2} + \pi n\right)}\right],$$

$$\lambda_{1n}^4 = \mu_{1n}^4 + 4\mu_{1n}^2\left[\frac{g_1(1)}{\theta_1} + \frac{B_0}{\theta_1^2 A_0} + \frac{C_0}{\theta_1^2 A_0}(-1)^{(n-1)}\right].$$

Taking into account μ_{1n} and $g_1(1)$ into the last equality, we'll get:

$$\lambda_{1n}^4 = \left(4n^4 + 8n^3 + 6n^2\right)\pi^4 i - \pi^2 n^2\left(\sin 1 - i(\cos 1 - 1) + 4i\frac{B_0 + (-1)^{n-1}C_0}{A_0}\right) + O(n),\ n \to +\infty.$$

After solution the equation $\Delta_k(\lambda) = 0$ $(k = 2,3,4)$ by the same way we'll get following asymptotic formulas:

$$\lambda_{2n}^4 = \mu_{2n}^4 - 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^{n-1}C_0}{\theta_1^2 A_0}\right]\mu_{2n}^2,$$

$$\mu_{2n} = \frac{(1+2n)\pi i}{2\theta_1},\ n \to -\infty,$$

$$\lambda_{3n}^4 = \mu_{3n}^4 + 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^n C_0}{\theta_1^2 A_0}\right]\mu_{3n}^2,$$

$$\mu_{3n} = \frac{(1+2n)\pi}{2\theta_1},\ n \to -\infty,$$

$$\lambda_{4n}^4 = \mu_{4n}^4 - 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^n C_0}{\theta_1^2 A_0}\right]\mu_{4n}^2,$$

$$\mu_{4n} = \frac{(1+2n)\pi i}{2\theta_1},\ n \to +\infty.$$

Substituting μ_{kn} $(k = 2,3,4)$ into equalities for λ_{kn}^4 $(k = 2,3,4)$ we'll get formula (10). The theorem is proved.

As it is known, that at equation (1) is parabolic in the sense of Shilov [10]. A following theorem allows us to find solution of the mixed problem (1)-(3) not only in case of parabolic in the sense of Shilov, but also wider classes:

Theorem 2. Suppose ,that function $\varphi(x)$ are satisfies to a following conditions $\varphi(x) \in C^2[0,1]$, $\varphi(0) = \varphi(1) = \varphi'(0) = \varphi'(1) = 0$ .If $A_0 \neq 0$, coefficients of the boundary conditions are complex numbers and $\operatorname{Im} \frac{B_0 \pm C_0}{A_0} \leq \frac{1}{4} \sin 1$, then mixed problem (1)-(3) has following solution

$$u(x,t) = -i \sum_{k=1}^{4} \sum_{n=1}^{\infty} \underset{\lambda=\lambda_{km}}{res}\, \lambda^3 e^{\lambda^4 t} \int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi , \tag{12}$$

here $G(x,\xi,\lambda)$ is a Green function of the corresponding spectral problem , λ_{kn} $(k = \overline{1,4}; n = 1,2,3,...)$ are all zeroes of the characteristic determinant $\Delta(\lambda)$.

Proof. Let's search solution of the mixed problem (1)-(3) as follows

$$u(x,t) = \sum_{k=1}^{4} \sum_{n=1}^{\infty} \underset{\lambda=\lambda_{km}}{res}\, \lambda^3 \int_0^1 G(x,\xi,\lambda) z(\xi,t,\lambda)d\xi . \tag{13}$$

Taking into account (12) into (1) and (2) we can find function $z(\xi,t,\lambda)$ in such form

$$z(\xi,t,\lambda) = -ie^{\lambda^4 t}\varphi(\xi)..$$

Taking into account the last into (13), we will get formula (12).

From condition $A_0 \neq 0$ can be said, that problem (4), (5) is regular [4,5]. It means, that out of $\delta > 0$ neighborhood of zeros of the characteristic determinant $\Delta(\lambda)$ inequality

$$\left|G^{(k)}(x,\xi,\lambda)\right| \leq \frac{M_k(x,\xi,\lambda)}{|\lambda|^{3+k}},\ k = \overline{0,3},\ \lambda \in S_j\ (j = \overline{1,8}) \tag{14}$$

is true, where $M_k(x,\xi,\lambda)$ are positive, bounded with respect to x and ξ functions and analytic function with respect to λ-complex parameter. At the same time, under condition $A_0 \neq 0$ and $\varphi(x) \in C^2[0,1]$, $\varphi(0) = \varphi(1) = \varphi'(0) = \varphi'(1) = 0$ for the function $\varphi(x)$ following formula of decomposition is true [5]:

$$\varphi(x) = -i \sum_{k=1}^{4} \sum_{n=1}^{\infty} \underset{\lambda=\lambda_{kn}}{res}\, \lambda^3 \int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi . \tag{15}$$

Taking into account (15) we can see that series given by formula (12) satisfies to initial condition. As the Green function $G(x,\xi,\lambda)$ is a solution of the homogeneous equation, corresponding to (4), (5), the series (12) formally satisfies to the boundary condition (2).

It is necessary for (12) and series, obtaining by differentiating it four times with respect to x, and ones with respect to convergence uniformly and absolutely. For that, taking into account conditions of the theorem and asymptotic of eigenvalues, defined by formula (10) we'll get:

$$\left|e^{t\lambda_{kn}^4}\right| = e^{t \operatorname{Re} \lambda_{kn}^4} = e^{-t\pi^2 n^2 \left(\sin 1 - 4 \operatorname{Im} \frac{B_0 \pm C_0}{A_0}\right) + O(n)} .$$

It shows, that if $A_0 \neq 0, \ \mathrm{Im}\frac{B_0 \pm C_0}{A_0} \leq \frac{1}{4}\sin 1$ formula satisfies according to Weierstrass theorem the functional series (12) uniformly and absolutely convergence. It means that our search formal operations are justified.

The theorem is proved.

REFERENCES

1. Ahmedov Saleh Z. Solution of a mixed problem for an equation of alternating type. Proceeding of Institute of Mathematics and Mechanics. Volume XXVI, (XXXIV), 2007. P.2-10
2. Birkhoff G.D. Boundary value problem and expansion problem of ordinary linear differential equations. Trans. Amer. Math. Soc., 1908, p. 373-395
3. Naimark M.A. Linear differential operators. Moscow, Nauka, 1969
4. Rasulov M.L. -Method of contour integration // M.- Science- 1964, -462 p.
5. Rasulov M.L. - Application of the residue method to the solution of the problem of differential equations // Baku, "Elm" Publishing house, 1989, 328
6. Rapoport I.M. On some asymptotic methods in the theory of differential equations. Publishing house of the Academy of Sciences of Ukraine, Kiev, 1954, 286 p.(in Russian)
7. Mamedov Yu.A.,Ahmadov S.Z-Study of the characteristic determinant related to the solution of the spectral problem // Bulletin of the Baku State University, series of physical and mathematical sciences. - 2005.- №2.- Pp.5-12
8. Mamedov Yusif A., Ahmadov Saleh Z. On solution of a mixed problemfor an equation of the fourth order with discontinuous coefficient. Transaction of National Academy of sciences of Azerbaijan, Baku-2006, Volume XXVI, №4, pp. 137-144
9. Sadovnichiy V. A, Lyubishkin V. A, Belabassy V.Yu. On zeros of entire functions of one class // Proceedings of the I.Q.Petrovsky seminar, Moscow , 1982, Issue 8, pp. 211-217
10. Eidelman S.D. Parabolic systems, Moscow, «Nauka» publishing house, 1964, pp.443
11. Leiva, H.; Narvaez, M.; Sivoli, Z. Controllability of impulsive semi linear stochastic heat equation with delay. International Journal of Differential Equations, 2020, Volume 2020, 10 pages.
12. Gorodetskiy, V.V.; Kolisnyk, R.S.; Martynyuk, O.V. On a nonlocal problem for partial differential equations of parabolic type. Bukovinian Mathematics Journal, 2020, Volume 8(2), pp.24-39.

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ARCHITECTURE AND MODEL OF A SPECIAL-PURPOSE ELECTRONIC DOCUMENT MANAGEMENT SYSTEM WITH A DISTRIBUTED STRUCTURE

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ABSTRACT

The importance of forming and bringing the organization of databases of medical institution branches into a more suitable form in terms of their structure, accessibility and flexibility of applications, and the development of methods in this regard are presented in the article. Thus, having each branch's own database increases the flexibility of local queries, minimizes duplication of general data, and provides a structure for distributing certain data in a way that is specific to the functions and medical aspects of the branches. Many existing methods for distributed databases have been considered and the suitability of some of them for medical documents has been noted.

1. Introduction

The geographical location of medical institutions affects the structure of the databases they use [1]. As is known, branches of medical institutions differ from each other in terms of functionality. Thus, the equipment or laboratory analysis technique system available in branch X of a medical institution may not be available in its other branches. This leads to differences in the types of documents produced in each of them and necessitates the organization of the database content between branches in accordance with their functionality. On the other hand, in order to ensure the organization of each branch's own databases, it is also proposed to ensure their geographical organization. Thus, when a patient who applies to a branch applies to that branch for the second time, if the query for his previous information is made not from the center, but from the database of the branch where he is currently located, then the query result can be obtained faster [2]. On the other hand, collecting existing databases in one center and creating replication is considered an important factor in ensuring that it becomes a more sustainable system [3]. In such a case, it is more appropriate to create a database system where each branch has its own database, as well as a replication (copy) of the databases in each other branch in a single center (Figure 1).

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In distributed databases, data can be distributed geographically or functionally [4]. A distributed database system is a network of databases where data is stored in multiple physical or logical locations [5]. In general, when a query is received by the system, the response is generated across all databases or by routing it to specific databases depending on the type of query [4]. Database distribution allows for faster query execution and better error recovery by distributing large amounts of data across multiple physical databases [2,6].

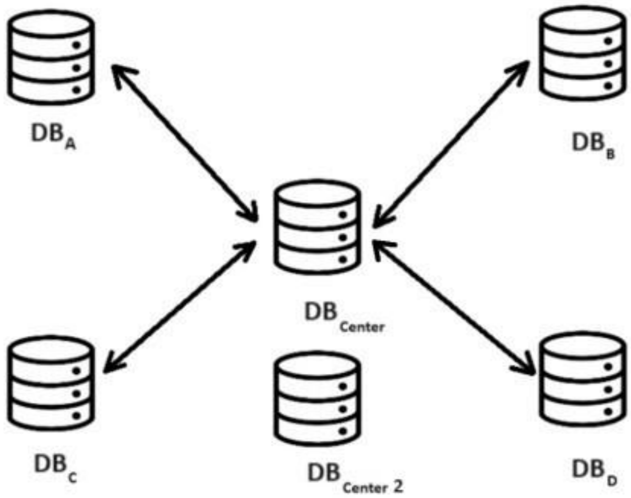

Figure 1. Organization of special-purpose distributed databases

There are two main concepts in distributed database management:

-Replication process

In this method, the data available in each database is copied to two or more different databases. If all the available data is copied to all databases, it is called a fully replicated system. Data replication is a method that increases the durability and consistency of databases. Database consistency is the fact that when users access any database node in the system, the same query produces the same result on each node. At the same time, replication is an effective factor in ensuring the security of databases. Thus, data deleted or lost in one database can be restored through other replications. The replication process ensures that data is constantly synchronized and copied with other databases [6].

Advantages:

- Minimizes loss by increasing overall data availability.
- It causes queries to be answered faster in the system, as queries are executed in parallel in distributed databases compared to a central database , so they are answered faster.
- When users send requests to different databases, the system becomes more resilient, as lost data is resynchronized and restored through other databases.
- It speeds up the process of reading from databases, as the request entering the system is redirected to the nearest node, minimizing network delays.

Disadvantages:

- The data in the databases must be constantly synchronized and updated, which increases the traffic load of the databases.

- Any changes made to any database node must be made to other databases as well, otherwise inconsistencies may arise between databases.
- Updating data causes delays, as each change is executed taking into account other databases.
- If the synchronization process is not performed correctly, inconsistencies arise between databases.
- Fragmentation process:

In this method, data fragments are distributed across different databases. When these fragments are combined, the original whole data is obtained again. Since the data is simply distributed in the fragmentation process, there is no consistency problem between them.

Advantages:

- It increases the productivity and performance of the system by allowing the parallel execution of multiple queries (especially for queries related to different fragments) [7].
- Storing the data that each center uses most frequently in databases located in close geographical locations leads to more efficient searches[2,4].
- Ensures that databases are more secure. As a result, if one node is attacked by hackers, not all of the data can be accessed, which ensures that the data is more secure overall [7].

Disadvantages:

- Distributing data across multiple databases and making queries on them more complex.
- The performance of applications depends on the speed of the network connecting the databases.
- Complexity of query optimization

The fragmentation process is divided into two parts in terms of the distribution of elements:

Horizontal fragmentation and vertical fragmentation. Horizontal fragmentation is basically the division of tables used in databases into rows and storing them in different databases (Figure 2). Vertical fragmentation is the process of dividing tables used in databases into columns or managing each table by storing it in a different database (Figure 3). To improve the performance of databases for enterprises, a hybrid distributed database structure is often used. Since both replication and fragmentation processes are used in databases, it allows ensuring the durability, consistency, and security of databases.

2. Building distributed databases for medical records

Databases for medical document databases should be structured as follows:

- The data generated in each branch should be stored in its own database. Thus, the data of those who apply to the branch is stored in the database of that branch and the results of the existing functions in each branch are stored in the database of that branch. This ensures both geographical and functional division of the databases.
- The data generated in each branch is also transmitted to the central database and copied there. The central database can be one or more than one. This structure makes the databases more secure by ensuring that they are more resistant to loss.

- During a search in one branch, information not available in the current database is generally queried through other branches or through the central database.

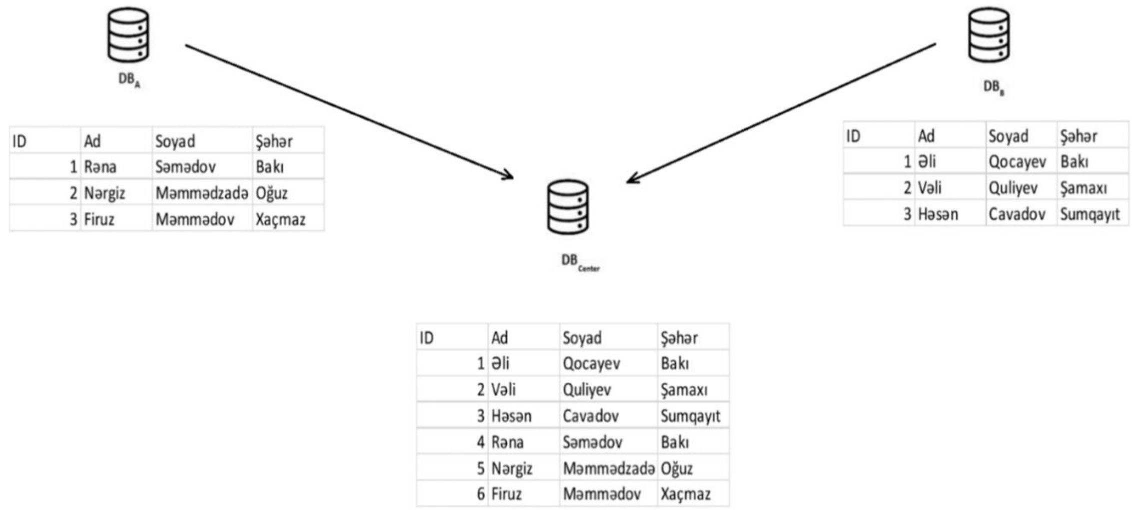

Figure 2. Database replication with horizontal distribution: how data within a table is distributed row by row and replicated at a central node

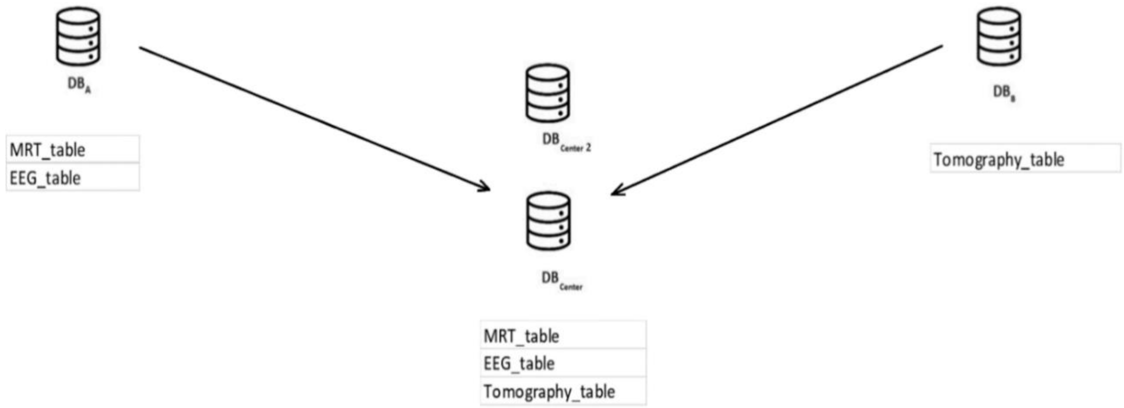

Figure 3. Database replication with vertical distribution: distribution of tables created in each node according to their functions across nodes and replication at the central node

3. Current methods used in database replication

The main issue in the replication process is the correct management of copying information generated in one database to other databases. Since a small error that may occur in the organization of work can lead to a violation of consistency between databases or a loss. There are two types of approaches to solving this problem:

1. Master-Slave approach or leader replication: Here, synchronization occurs by transmitting every change made on the master to the slave database(Figure 4). In this method, write operations are mostly performed on the master ,while read processes are performed from the slave databases [8].

2. Another replication management approach is leaderless replication, where each database is considered a master, where each database can perform read and write operations, and where each database communicates with other databases to ensure consistency among themselves[6,9].

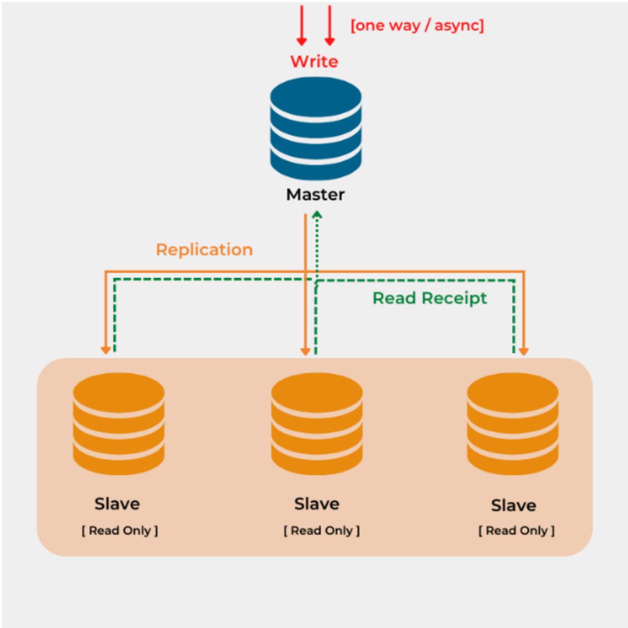

Figure 4. Master-Slave structured replication control method[11]

Both approaches have their own advantages and disadvantages:

Advantages of led replication:

1. Centralized management: The replication process is managed from a single center, which ensures easy management of requests and authorization processes.

2. Consistency: Leader-based replication is a very powerful method for ensuring consistency between databases, and in such systems, consistency is guaranteed to be high. Here, writes made to the leader are propagated to its followers, ensuring that all followers share the same data as the leader.

3. Simplicity: Configuration and management in leader-based replication are simpler than in the leaderless approach.

Advantages of leaderless replication:

1. High availability: In this method, since each of the system's databases participates in both write and read processes, the system's durability is significantly higher. Thus, in leader-managed replications, if the leader database fails, the system stops writing operations until another leader is elected.

2. High durability of database writes. In leaderless replication, the write process is more durable because it can be performed on any node in the system[9].

3. Load distribution: Through replication, data can be distributed to different geographical locations, so that requests to the system can be made over nearby locations, which supports increased performance[9].

There are many methods created to date for both approaches:

Several algorithms are used for data replication in distributed databases, and the choice of algorithm depends on factors such as consistency requirements, network conditions, and specific characteristics of the distributed system. Commonly used replication algorithms are as follows:

Primitive n -type replication:

In this approach, one node is designated as the primary node and the others are backup nodes. All write operations go to the primary node and the changes are then replicated to the backup nodes [8].

Voting-based Replication:

In this method, data written to one or more replicas is voted on among them to be replicated to other nodes. If there are n replicas, then data accepted by at least n/2+1 majority votes is written to the databases, achieving consistency[6].

Version vectors:

A version vector helps to know the versions of data stored in distributed databases and to track replication and consistency between them with these vectors. Along with consistency, it also helps to detect conflicts that may occur in the system[10].

Anti-Entropy Protocol:

This protocol performs periodic comparison and reconciliation of data between replicas to ensure consistency[6,11].

CRDTs (Conflict-Free Replication Data Structures):

CRDTs are data structures designed to be replicated across distributed nodes without the need for a centralized coordinator[12].

They ensure consistency by allowing simultaneous updates without conflicts.

4. **Algorithm for managing replications in special-purpose databases**

In the proposed model that ensures the distribution of medical documents between branches, the database located in each branch not only stores its own data, but also synchronizes with the central database, preventing data loss. In this system, the writing and reading process can be performed in each database. And this makes the system compatible with the leaderless replication model, despite the fact that it is synchronized with the central server. Here, a vector of versions that are interconnected with each other is used to manage replications.

Unlike special-purpose databases, let's initially look at the transaction version-based replication management method for managing n number of replications. Let's assume that each of the n number of databases performs the write process separately. Here, after each write operation, the changes must be synchronized with other databases. For the synchronization process, initially, each write transaction query rows (sql queries) are added to a table called “Transactions” in each database. The structure of the Transactions table reflects whether the rows are transactions and the columns are whether those transactions have been executed in n number of databases. And initially, all cells have the value 0. In whichever database the query is executed, the value 1 is written to the cell located at the intersection of the row and column corresponding to the query and that database in that table. If a database is down (down) and then restored (up), it takes the data in this Transactions table from other databases and combines them and executes them in the appropriate sequence on the current node. The advantage of this method is that the databases simply check the transaction tables in all databases with each other, which does not check whether all the data in each database is synchronized or not (as in the Snapshot method), which helps to perform the synchronization process more easily . Another issue in managing replications is the conflicts that may arise between them. Thus, when writing to the same data on two or more database nodes, conflicts arise between the values of the same data between these nodes, which leads to the problem of which data should be taken as the basis during synchronization. For managing replications, it is necessary to compare the synchronization status of transactions in the databases and to compare the versions of the changed data. Let's take a closer look at the above method:

Let's imagine that there are 4 replication databases in the system. A Tr_1 transaction is entered for a write operation to database 1. In this case, this transaction is initially added to the transaction table of the current database and, as soon as the transaction is executed here, it is assigned to the cell 1 located at the intersection of Db_1 and Tr_1 in that table (where Db_n represents the database on node 1). Then, a synchronization request is sent from this database to other databases. If each of the remaining 3 databases is up, this Transaction is added to their tables and, as soon as it is executed there, it is assigned to the corresponding cell 1 in that table. During the execution process of 1 in each database, information about this is sent to other databases, and in each sent database, to reflect the execution of this Transaction in other databases, this transaction is assigned to the corresponding cell 1 in the Transaction table according to the database execution. And finally, whichever database executed the query last, after all databases have executed this query, this transaction row is deleted from all tables. Thus, only queries for which the synchronization process has not been completed remain in this transaction table.

		Database 1		
	Db1	Db2	Db3	Db4
Tr1	1	0	1	0
Tr2	1	0	0	0
Tr3				
Tr...				
Trn				

		Database 2		
	Db1	Db2	Db3	Db4
Tr1				
Tr2				
Tr3				
Tr...				
Trn				

		Database 3		
	Db1	Db2	Db3	Db4
Tr1	1	0	1	0
Tr2				
Tr3				
Tr...				
Trn				

		Database 4		
	Db1	Db2	Db3	Db4
Tr1				
Tr2				
Tr3				
Tr...				
Trn				

a) The state of the databases before synchronization

		Database 1		
	Db1	Db2	Db3	Db4
Tr1	1	1	1	1
Tr2	1	1	1	1
Tr3				
Tr...				
Trn				

		Database 2		
	Db1	Db2	Db3	Db4
Tr1	1	1	1	1
Tr2	1	1	1	1
Tr3				
Tr...				
Trn				

		Database 3		
	Db1	Db2	Db3	Db4
Tr1	1	1	1	1
Tr2	1	1	1	1
Tr3				
Tr...				
Trn				

		Database 4		
	Db1	Db2	Db3	Db4
Tr1	1	1	1	1
Tr2	1	1	1	1
Tr3				
Tr...				
Trn				

b) State of databases after synchronization

Figure 5. Description of the method of leaderless management of replications in a distributed database

Let's say Db_2 and Db_4 are down. A transaction enters Db_1 and is sent to the others for synchronization. In this case, the values in the Transaction table in Db_1 and Db_3 will be $Tr_1\{1,0,1,0\}$, respectively. Then, when Db_4 becomes down, another Tr_2 transaction enters the system. In this case, the only up database Db_1 will have $Tr_1\{1,0,1,0\}$, $Tr_2\{1,0,0,0\}$. Then, as soon as each down database becomes up, it synchronizes the Trs from all other databases to its Transaction table and, as each one is executed, it sends information about this to the other up databases as well as its own database. And thus, the databases synchronize with each other and after synchronization, the values corresponding to the transactions in each database take the status $Tr_1\{1,1,1,1\}$, $Tr_2\{1,1,1,1\}$. The worst case scenario that can occur in the transaction response process is that all down databases are up while other up databases are down. Then, when the databases are up, there is a problem of confusion about which transaction is executed in which sequence. To prevent this, the initial execution date for each transaction in the Transactions table (the date that reflects the first execution of the transaction in any of the databases) is recorded in front of each transaction in the Transactions table. When this situation arises, the transaction dates are executed sequentially, and consistency and correct sequence will be preserved. Another issue in the replication synchronization process is conflict management. If two databases perform a write operation on the same data at the same time, then during synchronization, there is confusion about which data will be taken as the basis and which will be copied to the others. In this case, this issue is solved to some extent with the execution date of the transactions. In this method, the execution date of the transactions on the same data is selected as the basis and synchronized with other databases. In the transaction table, if all values for each transaction are 1, then the information about the execution of that transaction is deleted from the transaction table in the database of each node. This process is checked in the database of the last synchronized node and sends information about the deletion to other databases. The advantage of this method is that the system is more durable, so even if only 1 of the n databases responds to the server, the system can continue to work. An example of the implementation of these processes is illustrated in Figure 5.

It was noted that in the database structure shown above for special-purpose databases, replications are performed only on the central database. Although the proposed synchronization process was considered for n number of databases , this method can also be easily applied to databases without a leader and replicated only on a central server.

5. Conflict management solutions in organizing leaderless replications

One of the main problems in managing leaderless replication is the occurrence of conflicts. Thus, if two different nodes perform operations on the same data at the same time, or if two different nodes make changes to the same data during a period of time when the connection with each other is lost, two different versions of the same data will exist when the connection is restored. Various methods have been implemented to prevent such situations to date.

If an internet connection is restored on a node that has lost connection to other nodes, the following methods are available to balance the difference between them:

1. **Last Write Wins**: This method provides authentication between nodes by providing a timestamp for each transaction that occurs, determining which transaction came last.

2. **Conflict-free Replicated Data Types**: A CRDT is a data structure that can successfully merge update operations.

3. **Manual intervention**: In some cases, conflicts between databases can be resolved by manually intervening by analyzing the error that occurred.

With the help of these methods, databases can be identified and consistency between them can be ensured.

In databases intended for medical documents, the use of time stamps is considered appropriate for resolving conflicts. Thus, during the execution of each transaction in each node, the IDs of the affected objects are collected in a table using triggers and stored in the nodes. Later, when the nodes exchange information about the transactions, the time stamp reflecting the transaction and the execution time of the transaction and the ID of the affected object are compared. To ensure the accuracy of the time stamp, each node must use online international time indicators and the time indicators must be synchronized at certain time intervals when there is an Internet connection. If two separate nodes make changes to an object with the same ID without informing each other, the first or the last one among them is selected and sent to all nodes, and the transaction that is not selected is rolled back on the node where it was executed. The algorithmic sequence of this process is as follows:

For simplicity, let's look at the synchronization process between two nodes, A and B:

1. Both nodes that are disconnected from each other retain the transactions executed on them until the synchronization process. $\sum Tr_i^A$ and $\sum Tr_j^B$

2. Between them is restored, the transactions in them are combined with hash-structured sets of object IDs: $Tr_{conflicts} = HashSet_{by\ Object\ ID}(\sum Tr_i^A + \sum Tr_j^B)$

3. $Tr_{conflicts}$The timestamp for each object is selected for each of the initial incoming requests, and the rollback process is performed on the database where the other request was executed, and the newly selected execution request is executed on all nodes.

As a result, existing methods for managing the load of distributed structured special-purpose databases, both fragmentation and functional and geographical distribution of data between databases, as well as organizing their replication to increase the durability of databases, were reviewed, and appropriate methodologies and algorithms were developed for such special-purpose databases.

Conclusion:

The article examines the advantages of organizing databases in a distributed manner for medical documents, and develops their structure in a manner appropriate to the purpose. The methods used for replication, fragmentation, and conflict resolution of databases, which are the main issues for distributed databases, are examined and developed in a manner appropriate for medical documents. The structures formed aim to increase system performance, increase file availability, and increase system security by minimizing losses.

REFERENCES:

1. Haux, R. (2006). Health information systems – past, present, future. *International Journal of Medical Informatics* , 75(3-4), 268-281. DOI: 10.1016/j.ijmedinf.2005.08.002 *(Health information systems, geographic presence)*
2. Özsu, MT, & Valduriez, P. (2011). *Principles of Distributed Database Systems* (3rd ed.). Springer. *(Distributed DB fundamentals, fragmentation, query speed, data locality)*
3. Tanenbaum, AS, & Van Steen, M. (2007). *Distributed Systems: Principles and Paradigms* (2nd ed.). Prentice Hall. *(Distributed systems, availability, fault tolerance, replication)*
4. Özsu, MT, & Valduriez, P. (2011). *Principles of Distributed Database Systems* (3rd ed.). Springer. *(Fragmentation types, replication basics, distribution strategies)*
5. Bernstein, PA, & Newcomer, E. (2009). *Principles of Transaction Processing* (2nd ed.). Morgan Kaufmann. *(Distributed DB definition, replication, consistency)*
6. Kleppmann, M. (2017). *Designing Data-Intensive Applications: The Big Ideas Behind Reliable, Scalable, and Maintainable Systems* . O'Reilly Media. *(Replication (with/without leader), consistency models, fragmentation (partitioning), conflict resolution (LWW, CRDT), quorum, anti-entropy, availability)*
7. Navathe, SB, Elmasri, R. (2016). *Fundamentals of Database Systems* (7th ed.). Pearson. *(Fragmentation, performance, security aspects)*
8. Kleppmann, M. (2017). Chapter 5: Replication. In *Designing Data-Intensive Applications* . O'Reilly Media. *(Master-Slave/Primary-Backup replication, advantages, limitations)*
9. Kleppmann, M. (2017). Chapter 5: Replication. In *Designing Data-Intensive Applications* . O'Reilly Media. *(Multi-Leader/Leaderless replication, advantages, conflicts, high availability)*
10. Parker Jr, DS, Popek, GJ, Rudisin, G., Stoughton, A., Walker, BJ, Walton, E., Chow, JM, Edwards, D., Kiser, S., & Kline, C. (1983). Detection of mutual inconsistency in distributed systems. *IEEE Transactions on Software Engineering* , SE-9(3), 240-247. DOI: 10.1109/TSE.1983.236733 *(Concept of version vectors) - Or Kleppmann (2017) explains this concept.*
11. https://diadem.in/blog/how-to-configure-mysql-master-slave-replication/
12. Shapiro, M., Preguiça, N., Baquero, C., & Zawirski, M. (2011). Conflict-free replicated data types. *Proceedings of the 13th International Symposium on Stabilization, Safety, and Security of Distributed Systems (SSS'11)* , 386-400. DOI: 10.1007/978-3-642-24550-3_29 *(Basics of CRDTs) - Or Kleppmann (2017) explains this concept.*

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ROBUSTNESS EVALUATION OF DCT-DWT BASED INVISIBLE IMAGE WATERMARKING UNDER COMMON IMAGE ATTACKS

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ABSTRACT

Invisible watermarking has become a vital technique in digital image security, allowing hidden data to be embedded without affecting visual quality. This study addresses the challenge of maintaining watermark imperceptibility while resisting common image distortions. The goal of this research is to assess the effectiveness of a hybrid watermarking method that combines Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT). In the proposed approach, the watermark is embedded in the DCT coefficients of the low-frequency DWT sub-band. Experiments were conducted in both RGB and YCbCr color spaces, with varying strength factors (alpha). The method demonstrated high imperceptibility, with SSIM values above 0.998, but limited robustness against JPEG compression, Gaussian blur, and noise. These findings highlight the need for more resilient hybrid methods

12. Introduction

Digital watermarking is a crucial technology in multimedia security, enabling copyright protection, authentication, and content tracking [1]. Invisible watermarking methods embed information into digital media such that it is imperceptible to the human eye but can be retrieved algorithmically. Transform-domain techniques, such as DCT and DWT, are widely used for their robustness and frequency localization properties.

In this paper, we investigate a combined DCT-DWT approach for invisible watermark embedding and evaluate its quality and robustness under standard image attacks.

13. Related Work

2.1 Discrete Cosine Transform (DCT)

The Discrete Cosine Transform (DCT) expresses a finite sequence of data points as a sum of cosine functions oscillating at different frequencies. It is widely used in image compression and watermarking because it concentrates most of the signal energy into a few low-frequency components. The 2D DCT for an image block is defined as [2]:

$$C(u,v) = \frac{1}{4}\alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y) cos\left[\frac{(2x+1)u\pi}{2N}\right] cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

Where $\alpha(u) = \begin{cases} \frac{1}{\sqrt{2}}, u = 0 \\ 1, u > 0 \end{cases}$

and $f(x,y)$ is the pixel intensity at position (x,y).

2.2 Discrete Wavelet Transform (DWT)

DWT provides a time-frequency representation of the signal by decomposing the image into sub-bands of different frequency resolutions. A single-level 2D DWT decomposes an image into four sub-bands: LL (approximation), LH (horizontal), HL (vertical), and HH (diagonal). The LL sub-band contains the most significant information and is commonly used for embedding watermarks. The inverse DWT reconstructs the image using these sub-bands [3].

2.3 Common Image Attacks

Watermarked images are subject to various distortions in real-world scenarios. JPEG compression introduces quantization errors, Gaussian blur reduces sharpness, and Gaussian noise adds pixel-level distortions. A robust watermarking technique must withstand these attacks while maintaining watermark recoverability [4].

14. Research Methodology

We implemented a DCT-DWT-based invisible watermarking framework in Python using OpenCV and PyWavelets. The original image is first decomposed using a single-level Haar DWT. The LL sub-band is then transformed using DCT, and the watermark is added to the DCT coefficients using an embedding strength factor (alpha). The inverse DCT and inverse DWT reconstruct the watermarked image. This process was applied in both RGB and YCbCr color spaces, with a special focus on the Y channel in the latter for better perceptual embedding.

To evaluate quality, we used PSNR, SSIM [5] , and MSE between original and watermarked images. For robustness, we extracted the watermark after applying JPEG compression, Gaussian blur, and Gaussian noise, and evaluated similarity to the original watermark.

15. Experimental Results

The original image used in this experiment is a photograph of a decorative architectural structure located in a vibrant urban setting. This image was chosen for its rich color distribution, sharp edges, and varied texture regions, which make it an ideal candidate for evaluating the imperceptibility of watermarking methods. The watermark is a centered, minimalistic photography logo with a monochromatic color palette and a combination of serif and handwritten fonts, accompanied by a vintage camera icon. Its clean design and high contrast allow effective embedding and clearer visual assessment upon extraction. The watermark was embedded using the DCT-DWT technique at an alpha value of 0.1 to demonstrate the method's visual transparency. Figure 1 presents the original image, the corresponding watermarked image, and the extracted watermark before any attack.

Fig. 1. Original image, the corresponding watermarked image, and the extracted watermark before any attack

Before diving into attack resilience, it is important to first assess the baseline image quality after watermark embedding. The following tables present SSIM, PSNR, and MSE values comparing the original and watermarked images at different alpha values. These metrics confirm that the watermarking process introduces minimal visual distortion, providing a solid foundation for further analysis under attack conditions

Table 1. Image Quality Evaluation (Original vs Watermarked)

Alpha = 0.1

Metric	Value
SSIM (avg over R,G,B)	0.9985
PSNR(db)	49.92
MSE	0.66

Alpha=0.3

Metric	Value
SSIM (avg over R,G,B)	0.9983
PSNR(db)	50.21
MSE	0.62

Alpha=0.5

Metric	Value
SSIM (avg over R,G,B)	0.9982
PSNR(db)	50.38
MSE	0.60

The quality evaluation results presented in this section demonstrate that the proposed DCT-DWT watermarking method maintains a high level of imperceptibility. For all tested alpha values (0.1, 0.3, and 0.5), the SSIM values remain above 0.998, indicating nearly perfect structural similarity between the original and watermarked images. Additionally, PSNR values exceed 49 dB, which reflects excellent preservation of visual quality, as values above 40 dB are generally considered imperceptible to the human eye. The corresponding MSE values are all below 1, further confirming minimal pixel-level distortion introduced during embedding. These metrics collectively validate the effectiveness of the method in preserving image quality, which is critical for practical watermarking applications

Figure 2 shows a visual summary of the SSIM, PSNR, and MSE values for the extracted watermark after applying JPEG compression, Gaussian blur, and Gaussian noise across different alpha values. The results clearly illustrate that although the watermark is successfully embedded with minimal impact on the original image, its robustness against attacks remains limited. The SSIM values remain extremely low across all scenarios, indicating a significant degradation of structural similarity post-attack. Similarly, PSNR values are consistently below 5 dB, suggesting severe loss in signal fidelity. These visual metrics confirm the vulnerability of the DCT-DWT watermarking method to common image distortions and highlight the need for enhanced strategies in future work.

Robustness Evaluation (Extracted Watermark vs Original Watermark)

Alpha = 0.1

Attack Type	SSIM	PSNR (dB)	MSE
JPEG Compression	0.0114	4.81	21506.69
Gaussian Blur	0.0077	4.73	21903.92
Gaussian Noise	0.0109	4.83	21385.50

Alpha = 0.3

Attack Type	SSIM	PSNR (dB)	MSE
JPEG Compression	0.0117	4.86	21235.48
Gaussian Blur	0.0077	4.71	21978.04
Gaussian Noise	0.0108	4.86	21230.79

Alpha = 0.5

Attack Type	SSIM	PSNR (dB)	MSE
JPEG Compression	0.0120	4.88	21116.44
Gaussian Blur	0.0076	4.71	22074.31
Gaussian Noise	0.0122	4.88	21152.10

Fig.2. Summary of the SSIM, PSNR, and MSE values for the extracted watermark after applying JPEG compression, Gaussian blur, and Gaussian noise

5.Conclusion

The DCT-DWT watermarking method provides excellent invisibility but shows weak robustness under common attacks. Future work should explore hybrid methods such as DWT-SVD, block-based redundancy, or error-correction techniques to improve resilience. Additionally, CNN-based classifiers can be integrated to detect and adaptively adjust embedding strategies depending on content and predicted attack vulnerability

REFERENCES

[1] A. Astridefi, R. A.G. Gultom, Y. D. W, Asnar, H.A. D. Rimbawa, Audio, Text, Image, and Video Digital Watermarking Techniques for Security of Media Digital, International Journal of Progressive Sciences and Technologies (IJPSAT) ISSN: 2509-0119, Vol. 42 No. 1 December 2023, pp. 389-398

[2] https://www.mathworks.com/help/vision/ref/2ddct.html

[3] P. Vashist, R.Singh, Dr. R. R. Arora, Digital Watermarking Using DCT & DWT Technique, International Journal of Research, Volume 04, Issue 06, Ma y 2017

[4] C. Song, S. Sudirman, M. Merabti, D. Llewellyn-Jones, Analysis of Digital Image Watermark Attacks, 2010 7th IEEE Consumer Communications and Networking Conference

[5] A. Horé, D.Ziou, Image quality metrics: PSNR vs. SSIM, 2010 International Conference on Pattern Recognition

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