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A NUMERICAL ANALYSIS OF A QUEUING-INVENTORY SYSTEM WITH CATASTROPHES

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ABSTRACT

A mathematical model of a queuing-inventory system (QIS) with catastrophes is built. Incoming customers form a Poisson flow with rate λ. The customer servicing time in the considered QIS is zero. The (S, Q) replenishment policy is used to increase the inventory level in the system. Here, S is the maximum storage size of the QIS, and $Q = S - s$ indicates the fixed size of the proposed order, $(s < S/2)$. Analytical formulas for the calculation of steady-state probabilities and performance measures of the system are found. Key performance measures include average inventory levels, reorder rates, and customer loss probability. The results of numerical experiments are given. These experiments are used to study the behavior of performance measures of the system versus initial parameters of the considered hypothetical model.

1. INTRODUCTION

In classic queuing-inventory systems (QIS), it is assumed that the period to sell the inventory to consumers is zero and that system inventories never perish. There are also systems with perishable inventories. These systems fall under two classes: systems whose stocks perish within a certain time interval, and systems whose stocks are destroyed as a result of a catastrophe. Systems whose stocks perish within a certain time interval have been widely studied in [1-5]. Systems whose stocks are destroyed as a result of a catastrophe one by one are relatively understudied, see [6-8]. Systems whose entire stocks are instantly destroyed as a result of a catastrophe and whose servicing time is zero are studied in [9, 10]. In [9], using the “Up to S” replenishment policy is proposed. In the “Up to S” replenishment policy, when the inventory level reaches a certain level s, $0 \leq s < S$, an order is sent to increase inventory, and when new stocks arrive, the inventory level of the system reaches the full storage capacity. In this paper, the parameter S will also indicate the maximum size of the system storage. In [10], the random replenishment policy was used to increase the inventory level in the system. In this replenishment policy, when the inventory level drops to zero, an order is sent to increase inventory and the size of the proposed order is a random variable. In this paper, a similar model is studied using the (S, Q) replenishment policy, where $Q = S - s$ indicates the constant size of the proposed order.

2. DESCRIPTION OF THE MODEL AND PROBLEM STATEMENT

The maximum storage of the QIS under investigation is S and customers in it form a Poisson flow with rate λ. The order servicing time in the considered QIS is zero, i.e., it is a self-service system. Each customer receives a unit size inventory. If the inventory level is zero at the moment of arrival of a customer, it is lost with unity probability. Catastrophes may happen in the system storage. It is assumed that the flows of these catastrophes are also Poisson flows with rate κ. As a result of catastrophes, all stocks are instantly destroyed. If the inventory level of the system is zero, catastrophes do not affect the operation of the system.

The (s, Q) policy is used to increase the inventory level of the system. This policy is defined as follows: when the inventory level of the system reaches s, a new order is sent and the size of the order sent is S-s. The parameter of order lead time has exponential distribution with rate ν. The problem is to find the steady-state distribution of the system under consideration and its following performance measures: the average rate of orders sent to increase the inventory (Reorder Rate, RR), average inventory size, S_{av}, and the probability of loss of customers (P_l).

3. SOLUTION

One of the possible scenarios of changes in the system inventory level is shown in Fig. 2. Here, t_k, k=1,2,... are the instants when orders arrive in the system, ω_k, k=1,2,... are the instants when stocks enter the system, τ_k, k=1,2 indicates the instants when catastrophes occur.

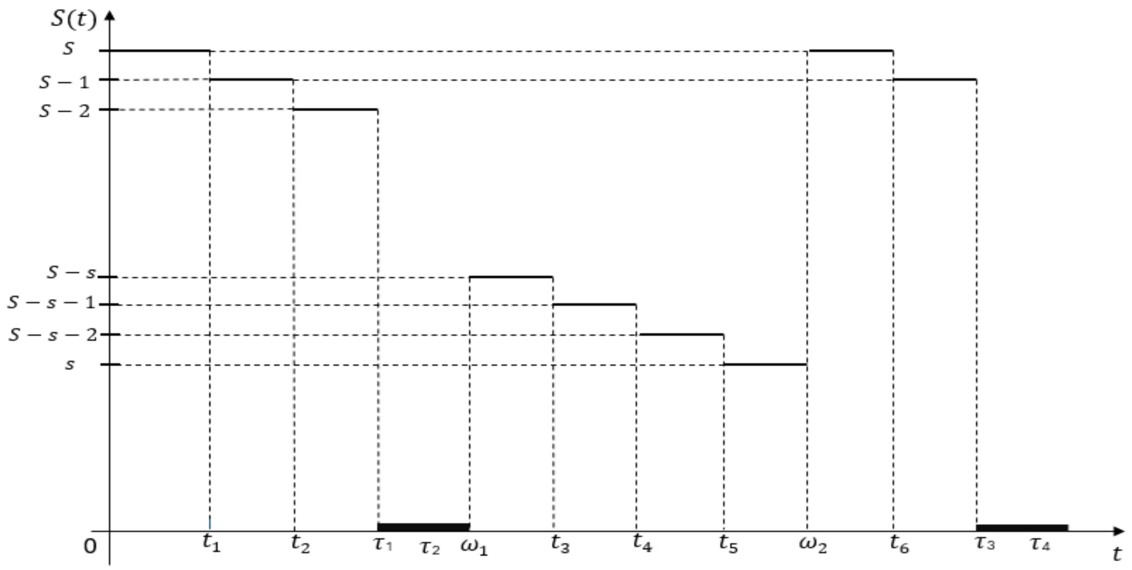


Fig. 1. A possible scenario of changes in the system inventory level

The state of the system at any time instant can be described by the inventory level. The inventory level of the system can take values $m = 0,1, \dots, S$, where S is the maximum size of the system storage. Since the flows (customers and catastrophes) entering the system are Poisson flows and the time required the stocks to enter the system obeys exponential distribution, we can say that the mathematical model of the system is a one-dimensional Markov chain. The state space of this chain is described by the set $E = \{0,1, \dots, S\}$.

The diagram of system states is shown in Fig. 2.

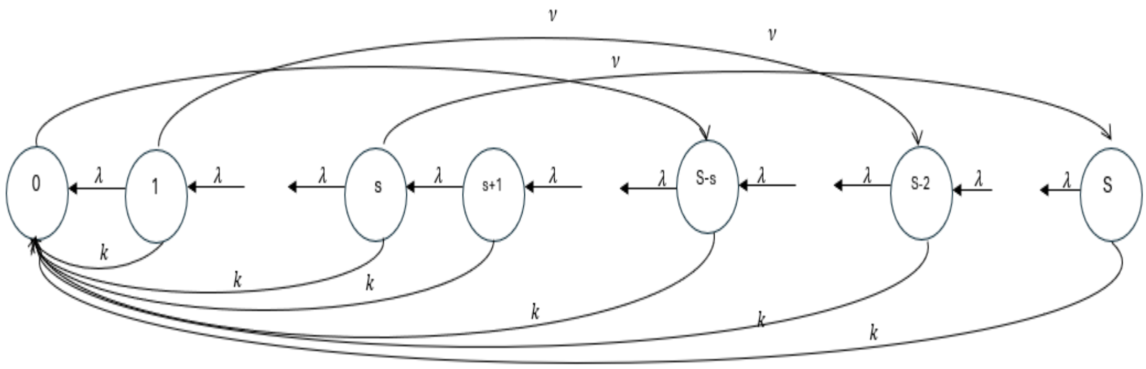


Fig 2. The diagram of system states

Denote the transition rate from a state m to a state m' by $q(m, m')$, $m, m' \in E$. Then the Markov chain generator is defined by the following formula:

$$q(m,m') = \begin{cases} \lambda \; if \; m > 1, m' = m-1 \\ \lambda + k \; if m = 1, m' = 0 \\ k \; if \; m > 1, m' = 0 \\ \nu \; if \; 0 \leq m \leq s, m' = m + S - s \end{cases} \tag{1}$$

The finite Markov chain under investigation has a steady state, that is, it is irreducible. We denote the probability of the system being in a state $m \in E$ by $p(m)$. Then the equilibrium equations for steady-state probabilities according to formula (1) are written as follows:

$$-\nu p(0) + (\lambda + k)p(1) + k\big(p(2) + \cdots + p(S)\big) = 0 \tag{2}$$

$$-(\nu + k + \lambda)p(m) + \lambda p(m+1) = 0, \quad 1 \leq m \leq s \tag{3}$$

$$-(k + \lambda)p(m) + \lambda p(m+1) = 0, \quad s + 1 \leq m \leq Q - 1 \tag{4}$$

$$\nu p(m - Q) - (\lambda + k)p(m) + \lambda p(m+1) = 0, \quad Q \leq m \leq S - 1 \tag{5}$$

$$\nu p(s) - (\lambda + k)p(S) = 0 \tag{6}$$

The following normalizing condition (7) is added to equilibrium equations (2)-(6):

$$\sum_{m=0}^{S} p(m) = 1. \tag{7}$$

According to equations (3)-(5), we can obtain the following formulas:

$$p(m+1) = a_{m+1}p(1), \quad a_{m+1} = (1 + \frac{(k+\nu)}{\lambda})^m, \quad 1 \leq m \leq s \tag{8}$$

$$p(m+1) = a_{m+1}p(1), \quad a_{m+1} = (1 + \frac{k}{\lambda})^{m-s}, \quad s + 1 \leq m \leq Q - 1 \tag{9}$$

$$p(m+1) = a_{m+1}p(1) - b_{m+1}p(0), \quad a_{m+1} = a_{s+1}\left(1 + \frac{k}{\lambda}\right)^{m-s} - \sum_{s=1}^{m-Q} a_s (1 + \frac{\nu}{\lambda})^{m-Q-s},$$

$$b_{m+1} = \frac{\nu}{\lambda}(1 + \frac{k}{\lambda})^{m-Q} \quad Q \leq m \leq S - 1 \tag{10}$$

Further, using equilibrium equation (2), we obtain the following formula for probabilities $p(0)$ and $p(1)$:

$$p(1) = p(0)\frac{k+\nu}{\lambda} - \frac{k}{\lambda} \tag{11}$$

Using normalizing condition (7) and formulas (8)-(11), we obtain the following formula for probability $p(0)$:

$$p(0) + p(1) + p(1)(a_2 + \cdots + a_{s+1}) + p(1)(a_{s+2} + \cdots + a_s) - p(0)\big(b_{Q+1} + \cdots + b_S\big) = 1$$

$$p(0) = \frac{1+\frac{k}{\lambda}\sum_{m=1}^{S} a_m}{1+\frac{k+v}{\lambda}\sum_{m=1}^{S} a_m - \sum_{m=Q+1}^{S} b_m} \tag{12}$$

After determining the steady-state probabilities, the performance measures of the system are calculated as follows.

- Average rate of orders sent to replenish the inventory (Reorder Rate, RR):

$$RR = \lambda p(s+1) + \kappa\big(1 - p(0)\big)\,. \tag{13}$$

- Average inventory size, S_{av}:

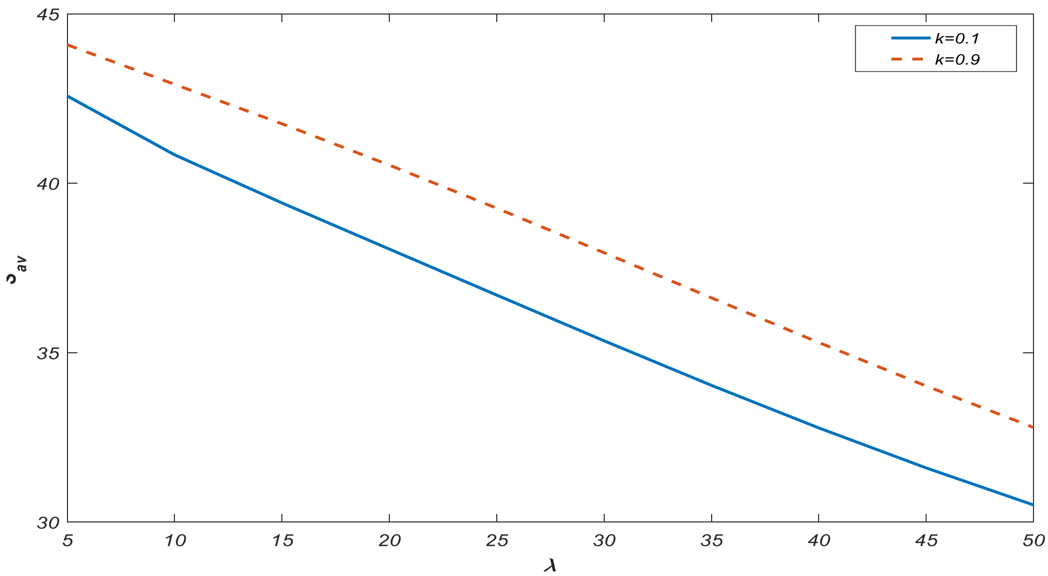
$$S_{av} = \sum_{m=1}^{S} mp(m) \tag{14}$$

- If at the moment of arrival of customers, the inventory level is zero, customers leave the system without receiving stocks. For this reason, the probability of loss, P_l, of customers is determined as follows:

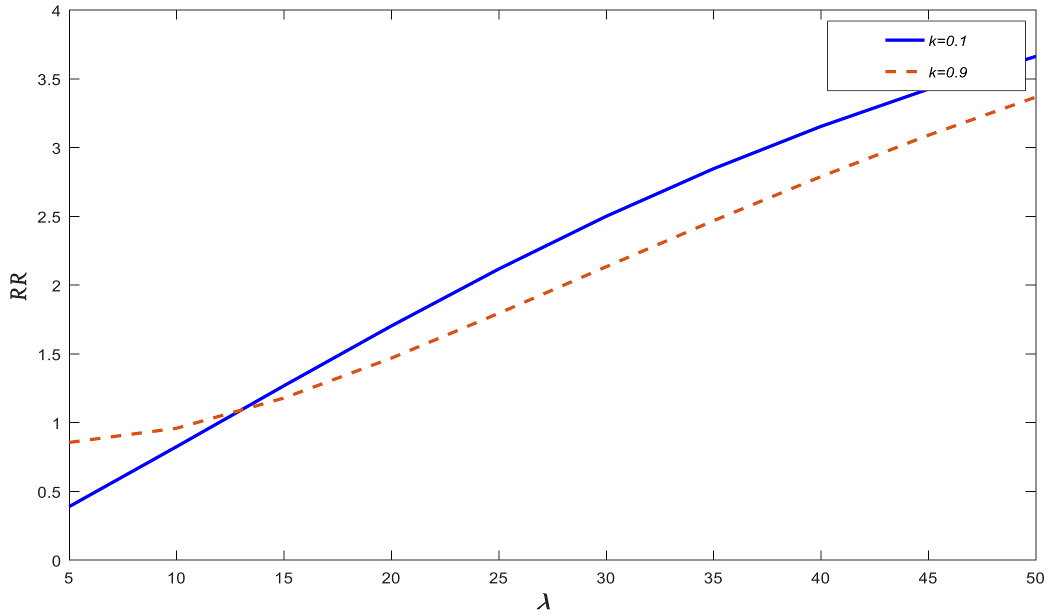
$$P_l = p(0) \tag{15}$$

4. NUMERICAL RESULTS

Using obtained formulas (8)-(10), experiments were conducted to calculate the performance measures of the system. In the following, we consider the results of these numerical experiments. The purpose of these experiments was to study the relationship between the performance measures of the system and its input parameters. The maximum system storage size in all the experiments is assumed to be constant, i.e., $S = 50$.



(a)



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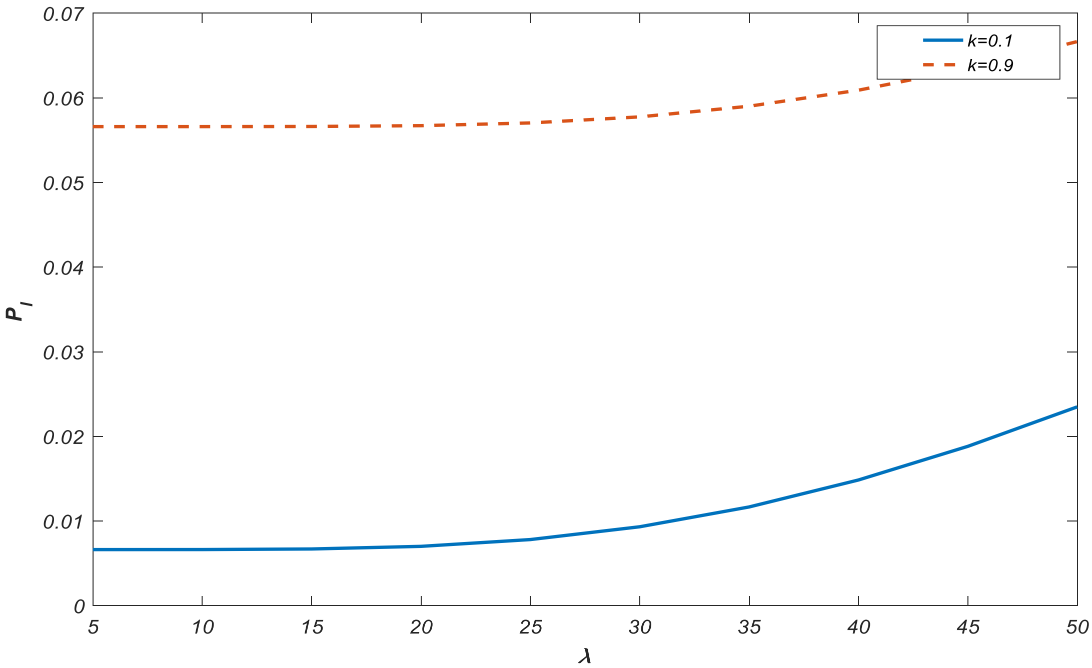
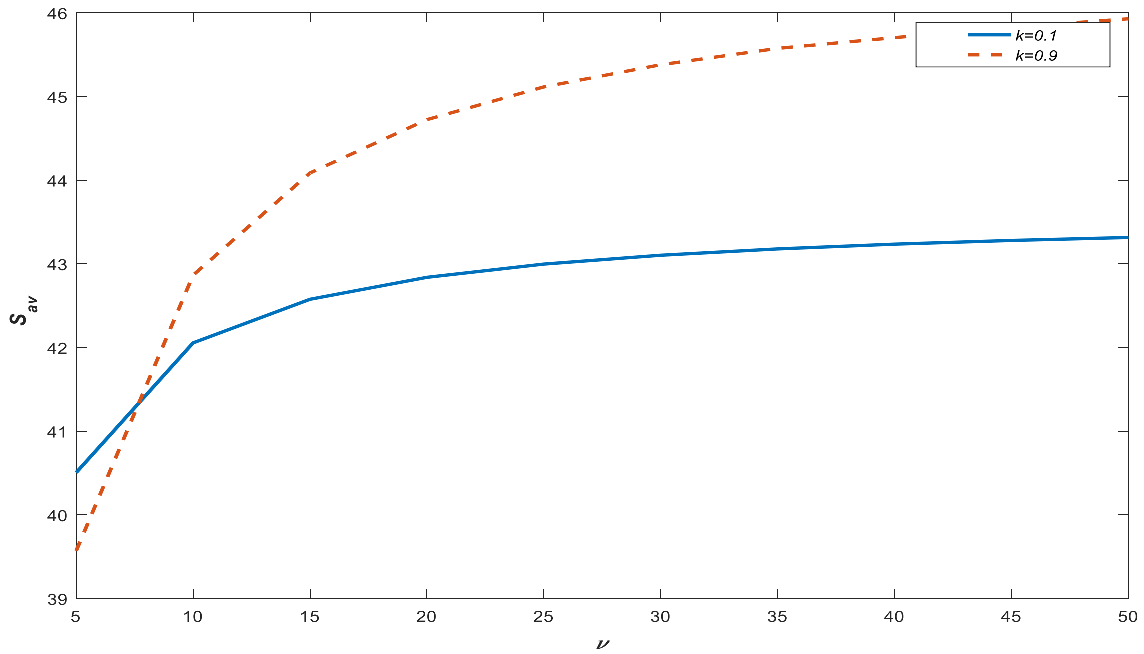


(c)

Fig.3. Performance measures vs λ;

Fig. 3 shows the graphs of the relationship between the performance measures of the system and the demand rate. As the demand rate increases, the average inventory level of the system decreases, see Fig. 3(a). Two different estimates of the catastrophe rate are examined in the graphs. The average inventory level of the system decreases relative to the change in the value of the catastrophe rate, see Fig. 3(a). When the value of the demand rate increases, the average rate of orders sent to replenish the system inventory (RR) increases; this is due to the fact that as the demand rate increases the inventory level rapidly drops to a critical level (s), see Fig. 3(b). Changes in the catastrophe rate have no significant effect on the average rate of replenishment orders sent, see Fig. 3(b). As the demand rate increases, the probability of order loss also increases, because the probability of the system inventory dropping to zero increases; as can be seen from the graph, as the catastrophe rate increases, the probability of order loss also increases, see Fig. 3(c).



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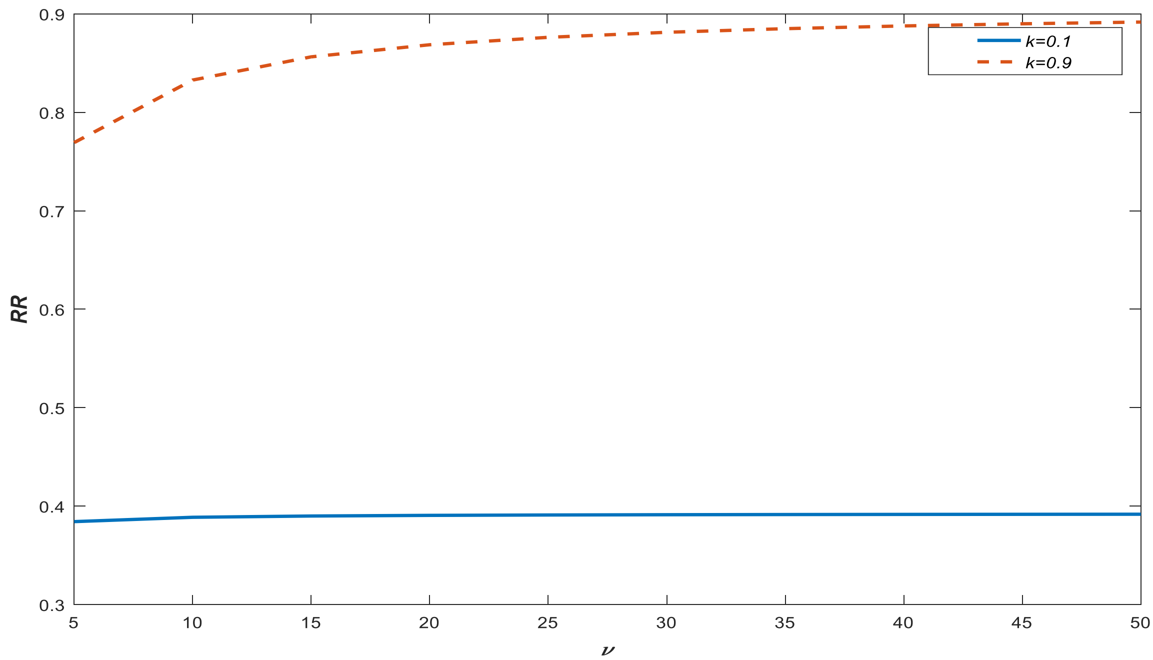

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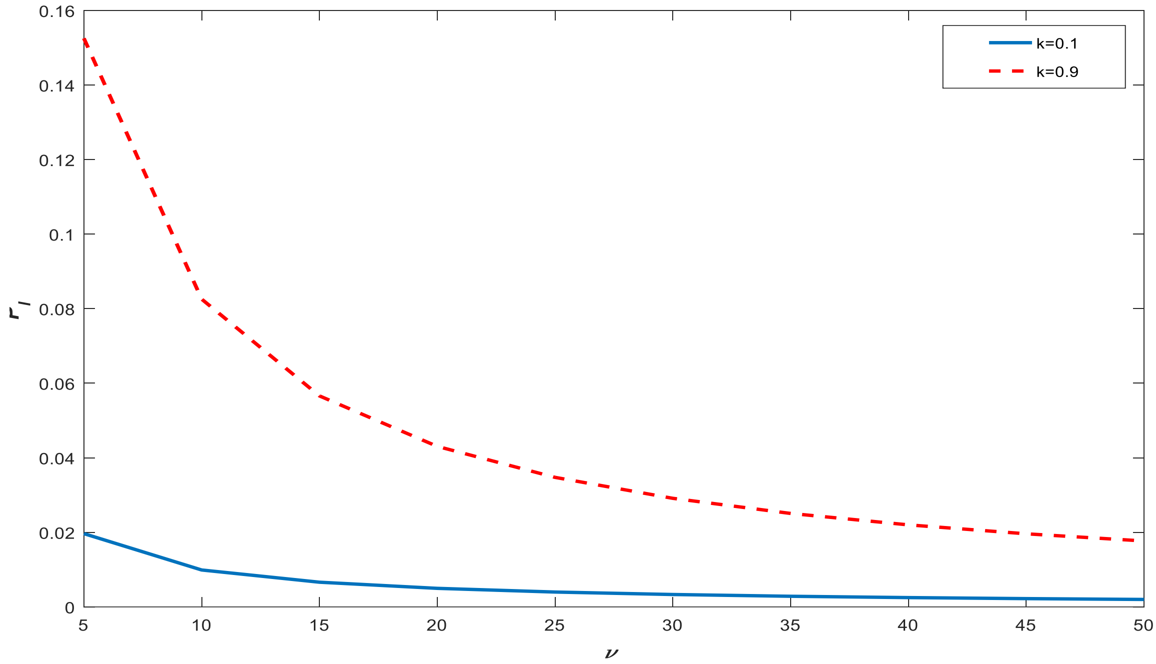
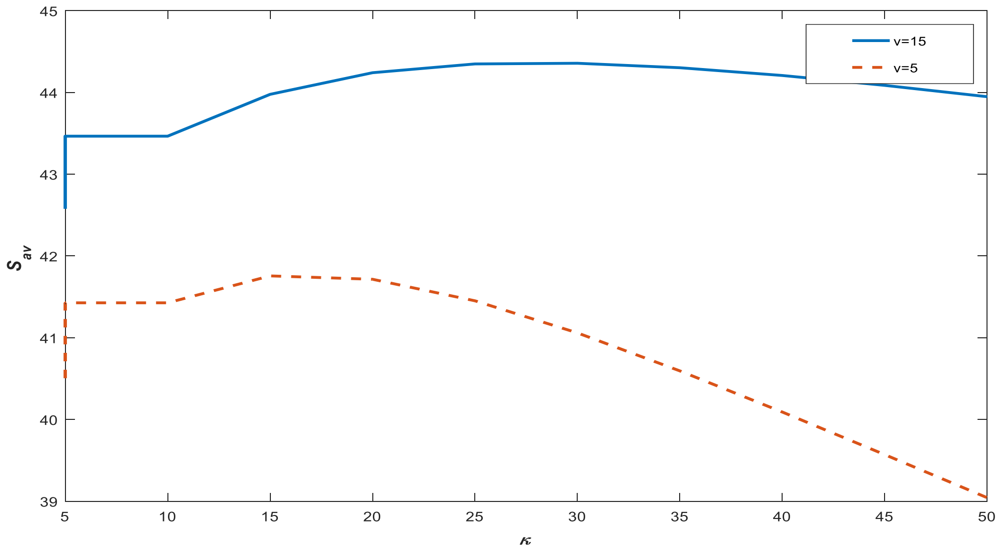


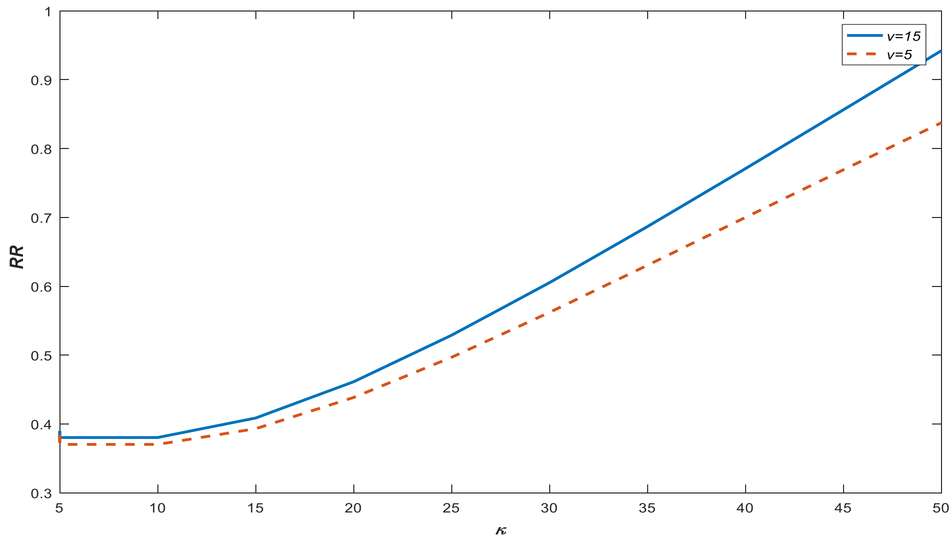
(c)

Fig.4. Performance measures vs ν;

Fig. 4 shows the relationship between the performance measures of the system and the inventory replenishment rate (ν). When the inventory replenishment rate increases, the average inventory level increases as well. At the same time, when the catastrophe rate increases, the probability of the system inventory dropping to zero increases, and because of this, the average inventory level decreases, see Fig. 4(a), for the selected initial values of the parameters, a twofold increase of the rate κ has no significant effect on the average level of such inventory. When the value of the inventory replenishment rate increases, the probability of the system inventory being non-zero increases, and because of this, the average reorder rate also increases, see formula (13). The average rate of orders sent to replenish the inventory increases relative to the variation of the catastrophe rate, and at small values of the parameter, changes in the catastrophe rate have no significant effect on the performance measure; however, at large values of the parameter, changes in the catastrophe rate have a significant effect on the performance measure, see Fig. 4(b). The decrease of the probability of customer loss is due to the fact that the probability of the system inventory dropping to zero decreases when the parameter increases; the performance measure increases relative to the variation of the catastrophe rate, which is particularly evident at small values of the parameter, see Fig. 4(c).



(a)



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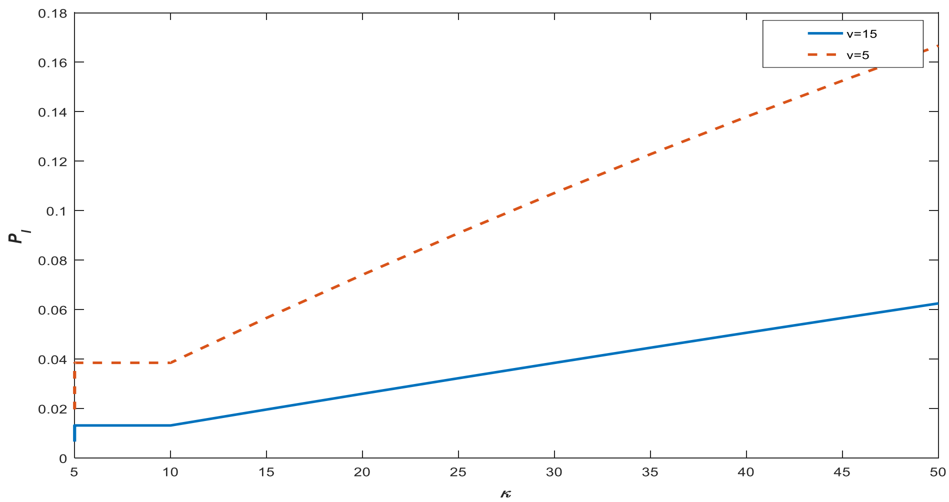

(c)

Fig.5. Performance measures vs κ;

Fig. 5(a) shows the relationship between the performance measures of the system and the catastrophe rate. As can be seen from the graphs, when the value of the parameter κ increases, the average inventory level of the system decreases. Different estimates of the inventory replenishment rate are examined in the graphs. For example, the average inventory level of the system increases relative to the inventory replenishment rate, see Fig. 5(a). When the value of the catastrophe rate increases, the inventory level of the system rapidly drops to zero, and for this reason, the average rate of orders sent to replenish the system inventory (see Fig. 5(b)) and the probability of customer loss (see Fig. 5(c)) increase. As can be seen from the graphs, changes in the inventory replenishment rate have a significant effect on the performance measures. Since at large values of the parameter κ the probability of the system inventory dropping to zero decreases, RR increases relative to the parameter, and P_l decreases; at large values of the parameter, changes in the inventory replenishment rate have a significant effect on the performance measures. See Fig. 5(a), Fig. 5(b).

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