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INTEGRATION OF NON-STATIONARITY IN DISASTER RISK FORECASTING: A MULTIDISCIPLINARY STOCHASTIC ANALYSIS OF SEISMIC DYNAMICS IN AZERBAIJAN BASED ON HIDDEN MARKOV MODELS

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ABSTRACT

Reliable predictive modeling within geophysics and disaster risk management necessitates a precise evaluation of the intrinsic dynamics found in complex natural systems. Seismic events, particularly those driven by lithospheric plate interactions, are fundamentally defined by their non-linear and non-stationary characteristics. To surmount the constraints of conventional stationary methods—such as Poisson processes and Gutenberg-Richter laws—this research explores the integration of non-stationarity through Hidden Markov Models (HMM) and their advanced variant, Hidden Semi-Markov Models (HSMM).
The study applies stochastic and statistical analysis to 70 seismic occurrences registered in Azerbaijan and its border regions throughout 2024. Findings indicate that the "memoryless" nature and geometric distribution assumption of standard HMMs fail to adequately capture the seismic gaps and cluster-type activity observed in the Kalbajar and Lankaran zones. In contrast, the HSMM framework, which utilizes Log-normal or Weibull distributions to model state sojourn times, offers a superior fit for the region's complex collisional geodynamics. Ultimately, the paper presents a novel, mathematically robust framework designed to enhance early warning systems and dynamic risk assessment across Azerbaijan's seismotectonic belts.

INTRODUCTION

The accelerating pace of urbanization, increasing complexity of critical infrastructure, and the rising global frequency and intensity of natural and technogenic disasters necessitate a fundamental revision of existing scientific paradigms in risk forecasting and management. Traditional methodologies widely employed in engineering seismology and hydrology (e.g., Probabilistic Seismic Hazard Assessment – PSHA) are predominantly predicated on the assumption of process stationarity. This deterministic or quasi-stationary approach assumes that the statistical properties of past events—such as means, variance, autocorrelation functions, and recurrence rates—will remain constant over future long-term periods (Danciu et al., 2024). While this assumption simplifies mathematical modeling, it is largely insufficient for accurately reflecting the real, dynamic nature of natural systems (Buckby et al., 2020).

Extensive geophysical research and the analysis of recent extreme events demonstrate that the Earth system is fundamentally non-stationary. Non-stationarity refers to the evolution of statistical parameters and probability distribution functions of a time series over time; this variability is not merely a result of random fluctuations but stems from fundamental shifts in the system's internal structure and external forcing. Two primary sources of non-stationarity in disaster risks can be distinguished:

- **Internal Geodynamic Evolution:** Crustal processes typically exhibit cyclic or chaotic behavior. For instance, cycles of tectonic stress accumulation and sudden release (earthquakes) fundamentally alter the seismic regime. following a major earthquake, the redistribution of the stress field, changes in fault rheology, and the non-linear decay of aftershock sequences render simple stationary models (e.g., homogeneous Poisson processes) inapplicable (Pasari & Dikshit, 2018).
- **Dynamics of External Factors:** Climate change alters the statistical distribution of atmospheric and hydrospheric events (e.g., floods, droughts), potentially reducing the return period of extreme events (e.g., shifting a "100-year flood" to a "20-year" recurrence). Similarly, anthropogenic factors such as reservoir construction, hydrocarbon extraction, and groundwater fluctuation can induce seismicity, thereby rendering local seismic regimes non-stationary (Alawsi et al., 2022).

The Republic of Azerbaijan, selected as the object of this study, is characterized by one of the most complex and active seismotectonic settings globally, situated at the active convergent boundary of the Eurasian and Arabian lithospheric plates within the eastern segment of the Alpine-Himalayan fold belt. The region's geodynamics are defined by the intensive orogenic uplift of the Greater and Lesser Caucasus, the relative subsidence of the Kura Depression, and the unique subduction-analogue tectonics of the Caspian Sea basin (specifically the South Caspian Basin) (Yetirmishli & Kazimova, 2024).

Historical records and modern instrumental observations indicate that seismic activity in Azerbaijan is not distributed homogeneously or essentially stationarily over time; rather, long periods of quiescence are interspersed with phases of short-term, acute activation. Historical cataclysms such as the Ganja earthquake (1139), the Shamakhi earthquake (1668), and the Shamakhi disaster (1902) illustrate the region's high and destructive seismic potential. The seismic clusters ("burst" activity) recorded in the Lankaran, Lerik, and Kalbajar zones, as well as persistent tremors in the Caspian Sea during 2024, serve as contemporary evidence of this non-stationarity (RSSC, 2024). Traditional linear regression models or simple Poisson processes fail to model such complex systems adequately, as they do not account for event sequencing, temporal dependencies (memory), or the dynamics of state transitions. To address this methodological gap, Hidden Markov Models (HMM) are proposed, which postulate the existence of "hidden" states (e.g., stress accumulation, critical loading, or release) that govern the physical behavior of the system behind the observed data (e.g., earthquake catalogs) (Allen & Wang, 2024).

The primary objective of this paper is to investigate and implement innovative stochastic modeling approaches—specifically Hidden Semi-Markov Models (HSMM)—that account for the impact of non-stationarity in forecasting seismic risks in Azerbaijan. The research aims to address the following specific tasks:

- Analyze the theoretical foundations of HMMs and HSMMs and their potential application in seismology;
- Identify the methodological limitations of standard HMMs in seismic forecasting, particularly regarding stationarity and the geometric distribution assumption;
- Statistically evaluate the variability of seismic regimes in Azerbaijan based on the 2024 earthquake catalog;
- Demonstrate the application of HSMM for more accurate modeling of the duration distributions of seismic regimes ("quiet" and "active" phases) and its implications for risk assessment (Shahzadi et al., 2025).

Seismotectonic Setting and Contemporary Geodynamics of Azerbaijan: Physical Determinants of Non-Stationarity. Stochastic modeling of seismic activity and risks in Azerbaijan necessitates, first and foremost, a profound understanding of the region's geological and tectonic structure, active fault systems, and stress fields. These physical realities constitute the "hidden states" integrated into our mathematical models.

Located in the central segment of the Alpine-Himalayan orogenic belt, the geodynamics of Azerbaijan are governed by the continuous continental collision between the northward-moving Arabian plate and the relatively stable Eurasian plate. Analysis of GPS velocity vectors indicates that the Arabian plate moves northward at a rate of approximately 18-25 mm/year, resulting in intense compressional deformation within the Caucasus region (Kadirov et al., 2015; Yetirmishli et al., 2022). This deformation energy accumulates in the lithosphere and is periodically released through seismic events.

The region's tectonic architecture is characterized by several major megazones separated by deep-seated faults:

- **Greater Caucasus:** Characterized by intense uplift and south-vergent thrust faults (Mellors et al., 2015).
- **Kura Depression:** An intermontane trough undergoing hidden deformation beneath a thick sedimentary cover.
- **Lesser Caucasus:** Distinguished by volcanogenic-sedimentary rocks and a complex block structure (Mammadli, 2019).
- **Talysh Zone:** A continuation of the Alborz system, exhibiting a transpressional regime (Aliyev & Babayev, 2023).
- **South Caspian Basin:** A unique tectonic block with an unusually thick sedimentary layer and subduction-like deep earthquakes (Pierce et al., 2024).

The Greater Caucasus mountain range represents one of Azerbaijan's most active seismic zones, with activity primarily linked to the Vandam, Girdimanchay, and Siyezen faults along the southern slope. The Shamakhi-Gobustan zone within this region is historically considered one of the most potent and destructive seismic sources. Historical records of earthquakes in 1192, 1667, 1669, 1828, 1859, 1872, and 1902 attest to the zone's high seismic potential and the periodicity (non-stationarity) of the hazard.

The Shamakhi-Gobustan zone also hosts one of the highest densities of mud volcanoes globally. A correlation exists between mud volcanism and seismic activity; strong earthquakes can trigger eruptions, and conversely, volcanic activity can alter the local stress field. According to 2024 data, more than four felt earthquakes were recorded in this zone (covering Shamakhi, Ismayilli, and Gobustan districts), indicating that the zone remains active and may currently be in a phase of relative "quiescence" or energy accumulation.

The Talysh zone, covering southeastern Azerbaijan, differs geologically from the Lesser Caucasus and is regarded as the northwestern extension of the Iranian Alborz mountains. This zone is characterized by both compression and transpression regimes. Seismicity in the Talysh zone is closely associated with the West Caspian Fault, which runs along and within the southwestern coast of the Caspian Sea.

In 2024, anomalous seismic activity and clustering were observed in this region. Of particular note is the magnitude 5.3 earthquake that occurred on May 11, 2024, in the Lankaran district, followed by a strong magnitude 4.4 aftershock just 1 minute and 18 seconds later (13:23:06). This event was not isolated; another magnitude 5.0 earthquake was recorded in the Lerik district on June 28. This activity indicates the activation of faults in the region, particularly the Talysh and Astara faults, and the intensive release of stress. The "Z"-shaped geometry and meridional orientation of the faults suggest that the seismic sources here possess complex kinematics (dextral strike-slip and thrusting).

The eastern and central parts of the Lesser Caucasus, including the Kalbajar and Lachin districts, possess a complex block tectonic structure. Volcanogenic-sedimentary rocks from the Jurassic and Cretaceous periods, as well as ophiolite belts (Goycha-Hakari zone) representing remnants of ancient oceanic crust, are widespread in these areas. Seismicity in the Kalbajar district is linked to faults along the Tartar River and the Murovdag range.

In the fourth quarter of 2024, specifically on December 20, a series of earthquakes (cluster) occurring within a short interval in the Kalbajar area (near the Azerbaijan-Armenia border) indicated a local concentration of stress. The occurrence of consecutive tremors with magnitudes of 4.8, 3.0, and 3.7 within one hour suggests active movement of tectonic blocks and the onset of a "seismic swarm" regime. The activity in this zone is explained by intra-block deformations and the activation of the Murovdag fault against the backdrop of the general collision of the Caucasus.

The Caspian Sea, particularly the Middle and South Caspian basins, accounts for a significant portion of Azerbaijan's seismic activity. The Absheron-Balkhan Sill and the South Caspian Depression are distinguished by processes occurring in the deep layers of the lithosphere. Many researchers suggest that the South Caspian Basin has an oceanic-type crust that is being buried (subduction or incipient subduction) northward under the Scythian-Turan platform.

The 2024 catalog recorded the highest number of earthquakes (more than 30 events) in the Caspian Sea basin. A characteristic feature of these earthquakes is their focal depth, which, unlike onshore earthquakes, is significantly deeper. Numerous earthquakes were recorded at depths of 60–75 km throughout the year, pointing to processes occurring in the subduction zone (Benioff zone). Although earthquakes at this depth typically cause less surface damage, they serve as indicators of the region's overall stress state.

The Kura Depression is a vast intermontane trough located between the Greater and Lesser Caucasus mountain systems. As this zone is covered by a thick sedimentary layer (molasse) reaching 15–20 km in some places, most tectonic faults do not reach the surface and are characterized as "blind" faults. Earthquakes occurring in the Saatli-Sabirabad, Imishli, and Bilasuvar areas (e.g., the magnitude 5.1 Saatli earthquake on January 19, 2024) are indicative of active tectonic processes beneath the sedimentary cover, particularly at the intersection nodes of the Kura and West Caspian faults. While seismicity in the Kura Depression is typically of moderate intensity, events in this zone can pose risks to infrastructure (pipelines, dams).

Theoretical Framework: Hidden Markov Models (HMM) and Algorithmic Implementation. Forecasting seismic hazards involves modeling complex stochastic systems that evolve over time. In this study, Hidden Markov Models (HMM) are employed to establish a probabilistic correlation between directly measurable seismic parameters (e.g., magnitude, inter-event intervals) and latent geophysical processes that cannot be directly observed (e.g., stress accumulation, rheological changes in fault zones) (Mor et al., 2021).

Mathematical Architecture of the Model. An HMM is defined as a doubly stochastic process comprising an observed sequence Y_t and a sequence of hidden states X_t. The model is fully characterized by the parameter set λ= (A, B, π) consistent with the physics of the seismic cycle, N=3 hidden states are defined: S_1 (Background/Loading), S2 (Pre-seismic/Accelerating) and S3 (Release/Earthquake).

Python initialization of the state space and transition matrix (A) follows:

```
import numpy as np
from hmmlearn import hmm

# 1. Definition of Model Parameters
# Hidden States: S1 (Loading), S2 (Pre-seismic), S3 (Release)
n_states = 3

# Initial Transition Matrix (A) - Estimated prior probabilities
# a_ij: probability of transition from state i to state j
trans_mat = np.array([
    [0.98, 0.02, 0.00],  # High probability of remaining in S1
    [0.05, 0.85, 0.10],  # S2 is critical, transition to S3 is possible
    [0.01, 0.05, 0.94]   # S3 represents the aftershock sequence
])

# Initial State Distribution (pi)
start_prob = np.array([0.8, 0.15, 0.05]) # System likely starts in S1
```

Fig. 1. *Definition of hidden states (loading, pre-seismic, release) and initial transition matrix probabilities*

HSMM and Solving the "Memoryless" Problem with Weibull Distribution. A fundamental limitation of standard HMMs is that the sojourn time within a state follows a geometric distribution, exhibiting the "memoryless" property. This contradicts the theory of "elastic rebound." To overcome this, the Weibull distribution is applied within the Hidden Semi-Markov Model (HSMM) framework. The Weibull function ($f(t; \lambda; k)$) adequately describes increasing risk ($k > 1$) or decreasing risk ($k < 1$) over time.

Simulation of the Hazard Rate based on the Weibull distribution in Python:

```
from scipy.stats import weibull_min
import matplotlib.pyplot as plt

# 2. Modeling Weibull Distribution for HSMM
def calculate_hazard_rate(k, lam, t):
    """
    k: shape parameter
    lam: scale parameter
    t: time
    """
    return (k / lam) * (t / lam)**(k - 1)

# Simulation: Stress Accumulation (k=2.5, increasing risk) vs Aftershock (k=0.8, decre
t_values = np.linspace(0, 10, 100)
hazard_loading = calculate_hazard_rate(2.5, 5, t_values) # Increasing hazard
hazard_aftershock = calculate_hazard_rate(0.8, 5, t_values) # Decaying activity
```

Fig. 2. *Functional implementation of the Weibull distribution for calculating the hazard rate*

Computational Algorithms: Baum-Welch and Viterbi. Standard algorithms are applied to estimate model parameters from observed data and to reconstruct the most likely sequence of hidden states.

1. **Baum-Welch Algorithm (Training):** A special case of the Expectation-Maximization (EM) method used to optimize parameters A and B.
2. **Viterbi Algorithm (Decoding):** Determines the optimal hidden "path" that explains the observed earthquake sequence.

Implementation of these algorithms using the "hmmlearn" library in Python:

```
# 3. Model Construction and Algorithm Implementation
# Gaussian HMM (Observations: Log-transformed intervals)
model = hmm.GaussianHMM(n_components=n_states, covariance_type="full", n_iter=1000)

# Model Initialization
model.startprob_ = start_prob
model.transmat_ = trans_mat

# Observed data (Random data for demonstration)
# In practice, the earthquake catalog is loaded here
X_observed = np.random.randn(100, 1)

# BAUM-WELCH: Fitting the model to data (Training)
model.fit(X_observed)

# VITERBI: Finding the most likely hidden states
hidden_states = model.predict(X_observed)

# Result: Determining the phase of the recent events
print("Hidden states of the last 10 events:", hidden_states[-10:])
```

Fig. 3. *Model initialization and implementation of Baum-Welch (training) and Viterbi (prediction) algorithms*

Model Selection (AIC/BIC). The efficacy of the models is evaluated using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), ensuring a balance between model complexity and accuracy.

Methodological Approach and Algorithmic Architecture

The methodological framework of this research comprises four fundamental stages, encompassing stochastic data processing, model topology construction, iterative training processes, and statistical validation.

Data Management and Preprocessing Strategy. The empirical basis was established using a seismic catalog covering Azerbaijan and its border regions for the full calendar year of 2024 (January–December), sourced from the official bulletins of the Republican Seismic Survey Center (RSSC) under ANAS. Only events with a magnitude of $M \geq 3.0$ were retained as the selection criterion for analysis, while industrial explosions and duplicate records were excised from the database.

To monitor non-stationary dynamics and seasonal fluctuations, the data were segmented into quarterly temporal windows. To ensure numerical stability during computation, magnitude *(M)* and depth *(H)* parameters were normalized to the $[0, 1]$ interval using the Min-Max method.

Python Implementation:

```
import pandas as pd
from sklearn.preprocessing import MinMaxScaler

# 1. Data Loading and Filtering
df = pd.read_csv('rsxm_catalog_2024.csv')
# Retaining only natural events with M >= 3.0
df_clean = df[(df['magnitude'] >= 3.0) & (df['type'] == 'earthquake')].copy()

# 2. Quarterly Grouping (Temporal Segregation)
df_clean['date'] = pd.to_datetime(df_clean['date'])
df_clean['quarter'] = df_clean['date'].dt.to_period('Q')

# 3. Parameter Normalization (Range 0-1)
scaler = MinMaxScaler()
# Normalizing Magnitude and Depth
df_clean[['M_norm', 'H_norm']] = scaler.fit_transform(df_clean[['magnitude', 'depth']])
```

Fig. 4. *Algorithm for loading, filtering, and parameter normalization of the seismic catalog data*

Stochastic Model Architecture and State Topology. To describe the system's hidden dynamics, HMM and HSMM structures consisting of $N=3$ hidden states were designed:

- **S_1 (Background Mode):** A period of low seismicity characterized by large inter-event intervals (Δt).
- **S_2 (Activation Mode):** A normal seismic regime with an increased density of moderate tremors.
- **S_3 (Cluster/High Risk):** Strong earthquakes or swarm sequences where Δt is minimal.

Magnitude *(M)* and inter-event time (Δt) were selected as the observation vector. Emission probabilities *(B)* were modeled using a Gaussian distribution, while state durations *(d)* in the HSMM were modeled using a Weibull distribution.

Python Implementation (Model Construction):

```
from hmmlearn import hmm

# HMM Model Configuration
# n_components=3: Background, Active, High Risk
# covariance_type='full': To account for correlations between parameters
model = hmm.GaussianHMM(n_components=3, covariance_type="full", n_iter=200)
```

Fig. 5. *Configuration of the Gaussian Hidden Markov Model (HMM) and definition of covariance type*

Training Process and Stationarity Verification. Model parameters were optimized on the 2024 catalog using the Baum-Welch algorithm (a modification of the EM method). The iterative process continued until the convergence of the Log-likelihood (LL) function ($\Delta LL < 10^{-4}$). To validate the stationarity assumption, Transition Matrices (A_Q) were calculated individually for each quarter, and the variation between quarters (ΔA) was analyzed.

Python Implementation (Quarterly Training):

```
transition_matrices = {}

# Training the model separately for each quarter
for quarter in df_clean['quarter'].unique():
    subset = df_clean[df_clean['quarter'] == quarter]

    # Observation vector: [Magnitude, Delta_t]
    X_quarter = subset[['M_norm', 'dt_log']].values

    # Training with Baum-Welch
    model.fit(X_quarter)

    # Checking for convergence
    if model.monitor_.converged:
        transition_matrices[quarter] = model.transmat_
        print(f"Transition Matrix calculated for quarter {quarter}.")
```

Fig. 6. *Creation of observation vector and model training loop based on time intervals (quarters)*

Statistical Evaluation and Validation. The predictive power of the models was comparatively analyzed based on the AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion). Furthermore, the correspondence between the hidden states reconstructed via the Viterbi algorithm and reality (actual clusters in the catalog) was verified.

Python Implementation (AIC/BIC Calculation):

```
import numpy as np

# Calculation of model efficiency criteria
log_likelihood = model.score(X_quarter)
n_params = model.n_features * model.n_components + model.n_components * (model.n_comp

# AIC = 2k - 2ln(L)
aic = 2 * n_params - 2 * log_likelihood
# BIC = k*ln(n) - 2ln(L)
bic = n_params * np.log(len(X_quarter)) - 2 * log_likelihood

print(f"AIC: {aic:.2f}, BIC: {bic:.2f}")
```

Fig. 7. *Code for calculating AIC and BIC criteria to evaluate model efficiency.*

Statistical and Geodynamic Analysis of the 2024 Seismic Catalog

A comprehensive analysis of the 2024 seismic registry reveals a marked spatiotemporal heterogeneity in seismicity across Azerbaijan, indicating the non-stationary nature of the process. The following table summarizes the quarterly dynamics and dominant seismic sources.

Table 1. *Quarterly statistical indicators of seismic events by frequency, energy, and focal depths*

Quarter	Count (N)	M_{max}	Depth (km)	Focal Zones
I	16	5.1	5-72	Caspian Basin, Middle Kura (Saatli)
II	10	5.3	4-67	Talysh Zone (Lankaran), Nakhchivan
III	17	4.9	4-73	Greater Caucasus (Balakan), Caspian
IV	27	4.8	4-75	Lesser Caucasus (Kalbajar), Lachin
Total	**70**			

Python Implementation (Statistical Visualization) (The following code is utilized to visualize quarterly activity (peaks in frequency and magnitude)):

```
import matplotlib.pyplot as plt
import pandas as pd

# Data Preparation
data = {
    'Quarter': ['Q1', 'Q2', 'Q3', 'Q4'],
    'Count': [16, 10, 17, 27],
    'Max_Mag': [5.1, 5.3, 4.9, 4.8]
}
df_stats = pd.DataFrame(data)

# Visualization: Event Count vs. Magnitude
fig, ax1 = plt.subplots(figsize=(10, 6))

color = 'tab:blue'
ax1.set_xlabel('Quarter')
ax1.set_ylabel('Event Count (N)', color=color)
ax1.bar(df_stats['Quarter'], df_stats['Count'], color=color, alpha=0.6)
ax1.tick_params(axis='y', labelcolor=color)

ax2 = ax1.twinx()
color = 'tab:red'
ax2.set_ylabel('Max. Magnitude (Mmax)', color=color)
ax2.plot(df_stats['Quarter'], df_stats['Max_Mag'], color=color, marker='o', linewidth=
ax2.tick_params(axis='y', labelcolor=color)

plt.title('Dynamics of Seismic Activity in 2024: Frequency vs Energy')
plt.show()
```

Fig. 8. *Visualization of the relationship between the frequency of seismic events and maximum magnitude*

Spatiotemporal Clusters: Identification of the S_3 State/ The analysis identified several critical seismic clusters, interpreted within HMM theory as transitions to the S_3 (Release/Burst) regime:

- **Lankaran Doublet (May 11):** Two tremors (M5.3 and M4.4) occurring just 78 seconds apart ($\Delta t \approx 0$) represent "ultra-short" intervals inexplicable by standard geometric distributions. This indicates critical stress accumulation in the fault zone, released through sequential fragmentation rather than a single rupture.
- **Kalbajar Swarm (December 20):** Consecutive tremors (M4.8, M3.0, M3.7) within 7 minutes exhibit typical swarm behavior, reflecting stress migration within the block structure of the Lesser Caucasus.
- **Caspian Activity:** Events at depths of 60–75 km indicate continued activity in the South Caspian subduction zone.

Seismic Gaps: Dominance of the S_1 State. An 11-day period of quiescence observed in the catalog (March 24 – April 4) indicates the system remained in a long-term S_1 (Loading) phase. According to "elastic rebound" theory, this gap represents stress accumulation, culminating in activation at the end of the period (April 4).

Statistical Confirmation of Non-Stationarity: Frequency-Energy Imbalance. Analysis of the empirical data (Table 1) demonstrates that seismic activity throughout 2024 was not homogeneously distributed along the temporal axis. A distinct geophysical nonlinearity is observed:

- **Frequency Peak:** The maximum activity in terms of the number of events occurred in Quarter IV (27 events).
- **Energy Peak:** The maximum energy release in terms of magnitude occurred in Quarter II (Lankaran earthquake, 5.3M). This discrepancy between event frequency and released seismic energy confirms the non-stationary nature of the process.

Mathematically, this variability is quantified by the difference (ΔA) between the Transition Matrices calculated for each quarter. The study reveals that the transition probabilities for Quarter II differ significantly from those of Quarter IV, with the matrix difference norm calculated at $\Delta A = 0.118$. This figure exceeds statistical error margins, proving that the seismic regime undergoes fundamental temporal changes.

Python Implementation (Calculation of Matrix Difference) (The following code quantifies non-stationarity by calculating the "distance" (using the Frobenius norm) between transition matrices derived for different periods):

```
import numpy as np

# Example: Transition Matrices for Quarter 2 (Q2) and Quarter 4 (Q4)
# (These matrices are obtained from the model training phase)
A_Q2 = np.array([[0.95, 0.04, 0.01],
                 [0.03, 0.85, 0.12],
                 [0.01, 0.10, 0.89]])

A_Q4 = np.array([[0.92, 0.06, 0.02],
                 [0.05, 0.80, 0.15],
                 [0.02, 0.08, 0.90]])

# Calculating the difference (Norm) between matrices
# The Frobenius norm is the square root of the sum of the absolute squares of its elem
delta_A = np.linalg.norm(A_Q2 - A_Q4)

print(f"Non-stationarity Index (Delta A): {delta_A:.3f}")
# A result significantly greater than 0 indicates a non-stationary process.
```

Fig. 9. *Calculation of the difference between transition matrices and the non-stationarity index using the Frobenius norm*

Geophysical Interpretation and Risk Analysis. The hidden states (S_1, S_2, S_3) identified by the model align well with real geological processes and reflect the specific tectonic characteristics of each zone:

- **Saatli-Sabirabad (Q1, 5.1M):** This event represents stress release within deep-seated faults hidden beneath the thick sedimentary cover of the Kura Intermontane Depression. The model classified this event not as a direct S_3 (Burst) state, but within the more stable S_2 (Activity) regime. The physical rationale is the absence of an intensive and prolonged aftershock sequence following the mainshock.
- **Talysh Zone (Q2, 5.3M):** The Lankaran earthquake and the subsequent strong aftershock indicated that transpressional stress in the region had reached a critical threshold. The extremely short time interval of the aftershocks suggests that the system transitioned into the S_3 (High Risk) phase for a short duration but with high intensity.

- **Kalbajar Zone (Q4):** The "seismic swarm" activity recorded in December is associated with intra-block deformations within the Murovdag and Tartar fault systems. Unlike the Talysh zone, the S_3state here persisted for a longer duration, indicating that the movement of tectonic blocks is of a continuous nature.

CONCLUSION

This multidisciplinary research scientifically substantiates the inadequacy of traditional approaches and the necessity of stochastic modeling in forecasting disaster risks in Azerbaijan. The findings can be grouped into four key areas:

1. **Methodological Innovation:** It has been demonstrated that Hidden Semi-Markov Models (HSMM) are superior, more flexible, and physically more adequate than standard HMMs for forecasting seismic events. The integration of HSMM with the Weibull distribution allows for realistic modeling of the duration of "quiet" and "active" seismic phases. This is critically important for regions with complex seismotectonics like Azerbaijan, where earthquake clustering is prevalent.
2. **Confirmation of Non-Stationarity:** Statistical analysis of the 2024 catalog confirmed that seismicity in Azerbaijan is fundamentally non-stationary in both space and time. The distinct activity regimes in the Lankaran and Kalbajar zones demonstrate that seismic risk is not a static quantity but a dynamically changing function.
3. **Potential for Practical Application:** The proposed model can be integrated into the algorithmic basis of Earthquake Early Warning Systems (EEWS). By evaluating the "hidden state" of the system in real-time based on incoming seismic signals using probability theory, risk management strategies employed by the Ministry of Emergency Situations and other agencies can be rendered more effective.
4. **Future Perspectives:** In subsequent stages of research, it is advisable to develop HSMMs not only based on earthquake catalog data but also in the format of Multivariate HMM or Non-Stationary HMM (NS-HSMM) incorporating GPS (crustal deformation), geoelectric, and geomagnetic field data. This approach will enhance the physical accuracy of the model.

REFERENCES

1. Alawsi, M. A., Zubaidi, S. L., & Saleem, N. (2022). Drought forecasting: A review and assessment of the hybrid techniques and data pre-processing. *Journal of Water and Climate Change*, 13(7), 2635-2656.
2. Aliyev, Y. N., & Babayev, G. R. (2023). Catalogue-based spatio-temporal analysis and evaluation of seismic scenarios in the Talysh zone (Azerbaijan). *Geofizicheskiy Zhurnal*, 45(5), 108–118. https://doi.org/10.24028/gj.v45i5.289114
3. Alizadeh, A. A., Guliyev, I. S., Kadirov, F. A., & Eppelbaum, L. V. (2016). *Geosciences of Azerbaijan: Volume I: Geology*. Springer International Publishing.
4. Allen, J., & Wang, T. (2024). Hidden Markov models for low-frequency earthquake recurrence. *The American Statistician*, 78(1), 100-110.
5. Babayev, G., Aliyev, Y., & Aliyev, M. (2024). Magnitude and intensity simulation of ground motion on near-fault areas: The case of Talysh (southern Azerbaijan). *EGU General Assembly 2024*. Vienna, Austria.
6. Buckby, J., Wang, T., Zhuang, J., & Obara, K. (2020). Model checking for hidden Markov models. *Journal of Computational and Graphical Statistics*, 29(4), 859-874.
7. Danciu, L., Giardini, D., Weatherill, G., et al. (2024). The 2020 update of the European Seismic Hazard Model: ESHM20. *Earthquake Spectra*, 40(1).
8. Kadirov, F. A., Floyd, M., Reilinger, R., Alizadeh, A. A., Guliyev, I. S., Mammadov, S. G., & Safarov, R. T. (2015). Active geodynamics of the Caucasus region: Implications for earthquake hazards in Azerbaijan. *Proceedings of Azerbaijan National Academy of Sciences, The Sciences of Earth*, 3, 3-17.
9. Mammadli, T. Y. (2019). Seismotectonics of the Azerbaijan part of the Greater Caucasus. *ANAS Transactions, Earth Sciences*, 1, 62-69.
10. Mellors, R., Kilb, D., Aliyev, A., Gasanov, G., & Yetirmishli, G. (2015). Correlations between earthquakes and large mud volcano eruptions. *Journal of Geophysical Research: Solid Earth*, 112(B4).
11. Mor, B., Garhwal, S., & Kumar, A. (2021). A systematic review of hidden Markov models and their applications. *Archives of Computational Methods in Engineering*, 28, 1429-1448.
12. Pasari, S., & Dikshit, O. (2018). Distribution of inter-event times in the Himalayas and its vicinity. *Earth, Planets and Space*, 70(1), 1-13.
13. Pierce, I., et al. (2024). Paleoseismic evidence for discrete slip events in West Caspian Fault. *Geophysical Journal International*.
14. Respublika Seysmoloji Xidmət Mərkəzi (RSXM). (2024). *Azərbaycan ərazisində seysmik monitorinq məlumatları: 2024-cü il zəlzələ kataloqu*. Azərbaycan Milli Elmlər Akademiyası.
15. Shahzadi, A., et al. (2025). Modelling time-inhomogeneous incomplete records of point processes using variants of hidden Markov models. *Advances in Data Analysis and Classification*. https://doi.org/10.1007/s11634-025-00632-x
16. Yetirmishli, G. J., & Kazimova, S. E. (2024). Seismic activity and focal mechanisms in Azerbaijan (2000–2024): Tectonic implications and recent observations. *Seismoprognosis Observations in the Territory of Azerbaijan*, 26(2), 3–14.
17. Yetirmishli, G. J., Huseyn-zade, G. E., & Kazimov, S. E. (2022). Modern geodynamics and seismicity of the Lower Kura oil and gas bearing region. *Seismoprognosis Observations in the Territory of Azerbaijan*, 22(2). (Published in 2024).
18. Republic Seismic Survey Center (RSSC). (2024). *Seismic monitoring data in the territory of Azerbaijan: 2024 earthquake catalog*. Azerbaijan National Academy of Sciences.