2025. Volume 9, Number 2 Pages 144-151

UDC:517.54

DOI: https://doi.org/10.30546/09090.2025.510.1023

MAJORIZATION RESULTS FOR A SUBCLASS OF MEROMORPHIC FUNCTIONS INVOLVING $q-$ LINEAR OPERATORS

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ARTICLE INFO

Article history
Received:2026-01-09
Received in revised form:2026-01-12
Accepted:2026-01-15
Available online

Keywords:
Meromorphic function;
Majorization;
$q-$ linear operators.

2010 Mathematics Subject Classifications: 30C45

ABSTRACT

In this work, we investigate several majorization results for a subordination class of meromorphic functions of complex order defined by a $q-$linear operator. The study focuses on establishing sufficient conditions under which majorization properties hold within this function class. By employing techniques from geometric function theory and operator theory, new inequalities and relationships are derived. Moreover, we present a number of new results that naturally arise from the main theorems and are stated explicitly in the form of corollaries. These corollaries highlight special cases and further implications of the obtained results. The findings contribute to the broader understanding of subordination, majorization, and $q-$calculus in the theory of meromorphic functions.

1. Introduction

Let M represent the class of meromorphic funtions f in the form of

$$f(z)=\frac{1}{z}+\sum_{n=0}^{\infty}a_n z^n, \quad (18)$$

which are analytic in the punctured disc $\mathring{\mathrm{U}}=\{z:0<|z|<1\}=\mathrm{U}/\{0\}$, where $\mathrm{U}=\mathring{\mathrm{U}}\cup\{0\}$. For the two functions $f(z)$ and $g(z)$ belonging to the class M, there exists a Schwartz function w, which is analytic in U with $|w(z)|\le|z|$ and $w(0)=0$, such that $f(z)=g(w(z))$, and the function f is subordinate to g, written as $f\prec g$. The following relationship holds if g is univalent:

$$f\prec g \Leftrightarrow f(0)=g(0), \text{ and } f\left(\mathring{\mathrm{U}}\right)\subseteq g\left(\mathring{\mathrm{U}}\right). \quad (19)$$

Because of its use in a variety of mathematical sciences, the study of $q-$ calculus (quantum calculus) has fascinated and motivated many scholars. One of the primary contributors among all the mathematicians who introduced the concept of $q-$ calculus theory was Jackson [1,2]. The formulation of this concept is widely used to investigate the natüre of different structures of function theory, such as $q-$ calculus was used in other branches of mathematics.

Though the authors of the first article [3] discussed the geometric nature $q-$ starlike functions, Srivastava [4] laid a solid foundation fort he use of $q-$ calculus in the context of function theory. Also, in [5], Srivastava provided a brief overview of basic or $q-$ calculus operators and fractional $q-$ calculus operators, as well as their applications in the geometric function theory of complex analysis. Later, the authors [6-8] investigated a number of useful properties for the newly defined $q-$ linear differential operator. While Srivastava et al. [9] introduced a generalized operator for meromorphic harmonic functions by using the idea of convolution.

Let $0<q<1$. For any nonnegative integer n, the $q-$ integer number n is defined by

$$[n]_q=\frac{1-q^n}{1-q}=1+q+q^2+...+q^{n+1},\ [0]_q=0.$$

In general, we will denote

$$[\delta]_q=\frac{1-q^{\delta}}{1-q},$$

for a noninteger number δ. Also, the $q-$ number shifted factorial is defined by

$$[n]_q!=[n]_q[n-1]_q...[2]_q[1]_q,[0]_q!=1.$$

Clearly,

$$\lim_{q\to1^-}[n]_q=n \text{ and } \lim_{q\to1^-}[n]_q=n!.$$

Let $a,q\in\mathbb{C}$ $(|q|<1)$ and $n\in\mathbb{N}_0=\mathbb{N}\cup\{0\}$. Then the $q-$ shifted factorial $(a;q)_n$ is defined by

$$(a;q)_0=1,\ (a;q)_n=\prod_{j=1}^{n}\left(1-aq^{j-1}\right),\ n\in\mathbb{N}.$$

Let $x\in\mathbb{C}-\{-n:n\in\mathbb{N}_0\}$. Then $q-$ gamma function is as follows:

$$\Gamma_q(x)=\frac{(q;q)_\infty}{(q^x;q)_\infty}(1-q)^{1-x},\ 0<q<1.$$

In geometric function theory, operators play an important role. Many authors present differential and integral operators, for example ([10, 11, 12, 13]). For a function $f\in\mathrm{M}$ given by (18), the $q-$ derivative (or $q-$ difference) of a function $f(z)$ is defined by [14,15].

$$\left(D_q f\right)(z)=\begin{Bmatrix}\dfrac{f(z)-f(qz)}{z(1-q)}, & z\neq0\\ f'(0), & z=0\end{Bmatrix} \tag{20}$$

provided that $f'(0)$ exists. We can easily observe from the definition of (20) that $\lim_{q\to 1^-}\left(D_q f\right)(z) = f'(z)$.

Suppose that $q\in(0,1)$, then $q-$ analog derivative of f as

$$\left(D_q f\right)(z) = -\frac{1}{qz^2} + \sum_{n=1}^{\infty}[n]_q a_n z^n. \tag{21}$$

Due to its use in numerous fields of mathematics and physics, the q−derivative operator Dq has fascinated and inspired many researchers. Jackson [16] was among the key contributors of all the scientists who introduced and developed the q−calculus theory. In 1991, Ismail [3] was the first to demonstrate a crucial link between geometric function theory and the q−derivative operator, but a solid and comprehensive foundation was provided in 1989 in a book chapter by Srivastava [5].

For $\varepsilon,\nu\geq 0,$ define the meromorphic q-analogue of Ruscheweyh operator $\mathrm{R}_{q,\nu}^{\varepsilon}:\Sigma\to\Sigma$ by Hadamard product (convolution)

$$\mathrm{R}_{q,\nu}^{m,\varepsilon} f(z) = \mathrm{R}_{q,\nu}^{m,\varepsilon}(z) * f(z) = \frac{1}{z} + \sum_{n=1}^{\infty}\left(\frac{[\varepsilon]_q + \nu\left([n+\varepsilon+1]_q - [\varepsilon]_q\right)}{[\varepsilon]_q}\right)^m a_n z^n \quad \left(r\in\square_0,\ 0<q<1\right), \tag{22}$$

where

$$\mathrm{R}_{q,\nu}^{m,\varepsilon}(z) = \frac{1}{z} + \sum_{n=1}^{\infty}\left(\frac{[\varepsilon]_q + \nu\left([n+\varepsilon+1]_q - [\varepsilon]_q\right)}{[\varepsilon]_q}\right)^m z^n,$$

was introduced and studied by Ekram et al. [17].

In 1967, Mac Gregor [18] introduced the Notion of majorization as follows.

Definition 1. Let complex-valued functions f and g be analytic in U. We say that f is majorized by g in U and write

$$f(z) \ll g(z) \quad (z\in \mathrm{U}) \tag{23}$$

if there exists a function $\varphi(z)$ (complex-valued function in U,) satisfying

$$|\varphi(z)|\leq 1 \text{ and } f(z) = \varphi(z)g(z) \quad (z\in\mathrm{U}). \tag{24}$$

Majorization (23) is closely related to the concept of quasi-subordination between analytic functions in U. Several researchers have published articles on this topic; for example, Tang et al. [19] gave the concept of majorization for subclasses of starlike functions based on the sine and cosine functions, Arif et al. [20] discussed majorization for various new defned classes, Cho et al. [21] obtained coefficient estimates for majorization, and Tang and Deng [22] defined the majorization problem connected with Liu-Owa integral operator and exponential function. This concept is also defned for $p-$valent function by Altintas and Srivastava [23] and for complex order by Altintas et al. [24].

The basic goal of this article is to examine and explain the idea of majorization in the context of the meromorphic function. Many researchers have shown their interest in this site. Goyal and Goswami [25, 26] studied this concept for majorization for meromorphic function with the integral operator, Tang et al. [19] discussed it for meromorphic sin and cosine functions, Bulut et al, Tang et al, and Janani [27, 28, 29] explained this concept for meromorphic multivalent functions, Rasheed et al. [30] investigated a majorization problem for the class of meromorphic spiral-like functions related with a convolution operator, and Panigrahi and El-Ashwah [31] defined majorization for subclasses of multivalent meromorphic functions through iterations and combinations of the Liu–Srivastava operator and Cho–Kwon–Srivastava operator and much more. In addition, there are several other articles on this topic [26, 32].

Here is the definition of our main function.

Definition 2. A function $f(z)\in \mathrm{M}$ is said to be in the class $\mathrm{M\ S}_{\mathrm{J}}^{m}(\varepsilon,\nu,q,\gamma)$ of meromorphic functions of complex order $\gamma\neq 0$ in $\overset{\circ}{\mathrm{U}}$, if

$$1-\frac{1}{\gamma}\left[\frac{zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)\right)}{\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)}+1\right]\prec\psi(z).$$

Now, we are going to choose a particular function instead of $\psi(z)$. This choice is

$$\psi(z)=\frac{1+Az}{1+Bz},\quad -1\leq B<A\leq 1,$$

and by applying the above-mentioned concept, we now consider the following class:

$$\mathrm{M\ S}_{\mathrm{J}}^{m}(\varepsilon,\nu,q,\gamma)=\left\{f(z)\in\mathrm{M}\ :1-\frac{1}{\gamma}\left[\frac{zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)\right)}{\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)}+1\right]\prec\frac{1+Az}{1+Bz}\right\}.$$

This class is related with well-known the Janowski class [33]. In the present work, we discussed majorization problem for the above-defined class of $\mathrm{M\ S}_{\mathrm{J}}^{m}(\varepsilon,\nu,q,\gamma)$.

2. Main Results

We state the following $q-$ analogue of the result given by Nehari [34] and Salvakumaran et al. [35].

Lemma 1. (See [36]) If the function $\varphi(z)$ is analytic and $|\varphi(z)|<1$ in U, then

$$\left|D_q\varphi(z)\right|\leq\frac{1-|\varphi(z)|^2}{1-|z|^2}. \tag{25}$$

Theorem 1. Let $-1\leq B<A\leq 1$, the function $f(z)\in\mathrm{M}$ and suppose $g\in\mathrm{M\ S}_{\mathrm{J}}^{m}(\varepsilon,\nu,q,\gamma)$ if $\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)$ is majorized by $\mathrm{R}_{q,\nu}^{m,\varepsilon}g(z)$ in $\overset{\circ}{\mathrm{U}}$, i.e.,

$$\mathrm{R}_{q,\nu}^{m,\varepsilon}f(z)\ll\mathrm{R}_{q,\nu}^{m,\varepsilon}g(z).$$

Then, for $|z| \leq r_1$,

$$\left|qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} f(z)\right)\right| \leq \left|qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)\right|, \tag{26}$$

where r_1 is the smallest positive root of the following equation:

$$\left(1-r^2\right)\left(1-\left|\gamma(A-B)+B\right|r\right)-2rq\left(1+|B|r\right)=0. \tag{27}$$

Proof. Since $g \in \mathrm{M}\,\mathrm{S}_{\mathrm{J}}^{m}(\varepsilon,\nu,q,\gamma)$, we readily obtained from Definition 2 that

$$1-\frac{1}{\gamma}\left[\frac{zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)}{\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)}+1\right] \prec \psi(z),$$

$z \in \overset{\circ}{\mathrm{U}}$ and

$$\psi(z)=\frac{1+Az}{1+Bz}.$$

By Lemma 1, there exists a bounded analytic function w in U and

$$1-\frac{1}{\gamma}\left[\frac{zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)}{\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)}+1\right]=\frac{1+Aw(z)}{1+Bw(z)}, \tag{28}$$

with $w(\infty)=\infty.$ From (28), we obtain

$$\frac{zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)}{\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)}=-\frac{\left[\gamma(A-B)+B\right]w(z)+1}{1+Bw(z)}, \tag{29}$$

where $w(z)$ is the well-known class of bounded anlytic functions in U such that

$$|w(z)| \leq |z| \quad (z \in U). \tag{30}$$

From (29) and making use of (30), we obtain

$$\left|\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right| \leq \frac{1+|B||z|}{1-\left|\gamma(A-B)+B\right||z|}\left|zqD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)\right|. \tag{31}$$

Since $\mathrm{R}_{q,\nu}^{m,\varepsilon} f(z)$ is majorized by $\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)$ in $\overset{\circ}{\mathrm{U}}$, from (24),

$$\mathrm{R}_{q,\nu}^{m,\varepsilon} f(z)=\varphi(z)\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z).$$

By applying $q-$ derivative on the previous equation write z as in [29] and then multiplying by qz, we have

$$\begin{aligned} qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} f(z)\right) &= qzD_q\varphi(z)\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)+qz\varphi(z)D_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right) \\ &= qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)\left[\varphi(z)+\frac{D_q\varphi(z)\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)}{D_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)}\right]. \end{aligned} \tag{32}$$

Noting that $\varphi(z)$ is the Schwartz function, so $\operatorname{Re}(\varphi(z))>0$ in $\mathring{U}$, $\varphi(z)\neq 0$ for all $z\in\mathring{U}$, satisfies the $q-$ analogue result given by [18] proved in Lemma 1.

Now, using (31) and (25) in (32), we have

$$\left|qzD_q\left(R_{q,\nu}^{m,\varepsilon}f(z)\right)\right|\leq\left|qzD_q\left(R_{q,\nu}^{m,\varepsilon}g(z)\right)\right|\left[\left|\varphi(z)\right|+\frac{1-\left|\varphi(z)\right|^2}{1-|z|^2}\frac{rq\left(1+|B||z|\right)}{1-\left|\gamma(A-B)+B\right||z|}\right].$$

Let us take $|z|=r<1$ and $\left|\varphi(z)\right|=\zeta,(0\leq\zeta\leq 1)$; we obtain

$$\left|qzD_q\left(R_{q,\nu}^{m,\varepsilon}f(z)\right)\right|\leq Y(r,\zeta)\left|qzD_q\left(R_{q,\nu}^{m,\varepsilon}g(z)\right)\right|.$$

We define

$$Y(r,\zeta)=\zeta+\frac{rq\left(1-\zeta^2\right)\left(1+|B|r\right)}{\left(1-r^2\right)\left(1-\left|\gamma(A-B)+B\right|r\right)},\quad\left(0\leq\zeta\leq 1,\ 0<r<1\right).$$

To determine r_1, it is sufficient to choose

$$r_1=\max\left\{r\in[0,1):Y(r,\zeta)\leq 1,\quad\forall\zeta\in[0,1]\right\},$$

equivalently,

$$r_1=\max\left\{r\in[0,1):Y^*(r,\zeta)\geq 0,\quad\forall\zeta\in[0,1]\right\},$$

where

$$Y^*(r,\zeta)=\left(1-r^2\right)\left(1-\left|\gamma(A-B)+B\right|r\right)-rq(1+\zeta)\left(1+|B|r\right).$$

Clearly, when $\zeta=1$, the above function $Y^*(r,\zeta)$ assumes its minimum value, namely,

$$\min\left\{Y^*(r,\zeta):\zeta\in[0,1]\right\}=Y^*(r,1)=\psi^*(r),$$

where

$$\psi^*(r)=\left(1-r^2\right)\left(1-\left|\gamma(A-B)+B\right|r\right)-2rq\left(1+|B|r\right).$$

Next, we obtained the following inequalities:

$$\psi^*(0)=1>0 \text{ and } \psi^*(1)=-2q\left(1+|B|\right)<0,$$

there exists r_1 such that $\psi^*(r)\geq 0$ for all $r\in[0,r_1]$, where r_1 is the smallest positive root of (27). The proof of Theorem 1 is completed.

Putting $A=1$ and $B=-1$ in Theorem 1, we get the following result.

Corollary 1. Let the function $f(z)\in M$ and suppose $g\in M\,S_J^m(\varepsilon,\nu,q,\gamma)$ if $R_{q,\nu}^{m,\varepsilon}f(z)$ is majorized by $R_{q,\nu}^{m,\varepsilon}g(z)$ in $\mathring{U}$, i.e.,

$$R_{q,\nu}^{m,\varepsilon}g(z)\ll R_{q,\nu}^{m,\varepsilon}g(z).$$

Then, for $|z| \leq r_2$,

$$\left|qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)\right| \leq \left|qzD_q\left(\mathrm{R}_{q,\nu}^{m,\varepsilon} g(z)\right)\right|,$$

where r_2 is the smallest positive root of the following equation $(1-r)(1-|2\gamma-1|r)-2rq=0.$

Conclusion

In this study, majorization results are obtained for a subclass of meromorphic functions defined by a $q-$ linear operator. The results generalize existing works and may lead to further studies through new subclasses or operators.

REFERENCE LIST

[1] Jackson, F. H. (1909). On $q-$ functions and a certain diference operator, Trans. R. Soc. Edinb., 46, 253–281.

[2] Jackson, F. H. (1910). On $q-$ definite integrals, Pure Appl. Math. Q., 41, 193–203.

[3] Ismail, M. E. H., Merkes, E. Styer, D. (1990). A generalization of starlike functions, Complex Variables, Theory and Appl.: An Int. J., 14, 77–84.

[4] Srivastava, H. M. (2020). Operators of basic (or q-) calculus and fractional q-calculus and their applications in geometric function theory of complex analysis, Iran J. Sci. Technol. Trans. A Sci., 44, 327–344.

[5] Srivastava, H. M. (1989). Univalent functions, fractional calculus, and associated generalized hypergeometric functions, in Univalent Functions, Fractional Calculus, and Their Applications, H. M. Srivastava and S. Owa, Eds., pp. 329–354, John Wiley, Sons, New York, NY, USA.

[6] Ahmad, B., Arif, M. (2018). New subfamily of meromorphic convex functions in circular domain involving q-operator, Int. J. Anal. Appl., 16, 75–82.

[7] Kota, W. Y., El-Ashwah, R. M., Breaz, N. (2025). Hadamard Product on Subclasses of Meromorphic Functions Involving q-Difference Operator. Journal of Function Spaces, 2025(1), 9959888.

[8] Arif, M., Haq, M. U., Liu, J. L. (2018). A subfamily of univalent functions associated with q-analogue of Noor integral operator, J. Funct. Spaces, 2018, 1–5.

[9] Srivastava, H. M., Arif, M., Raza, M. (2021). Convolution properties of meromorphically harmonic functions defined by a generalized convolution q-derivative operator, AIMS Mathematics, 6, 5869–5885.

[10] Abbas, M. I. (2020). Existence results and the Ulam stability for fractional differential equations with hybrid proportional Caputo derivatives, J. Nonlinear Funct. Anal. vol. 2020, Article ID 48.

[11] Libera, R. J. (1965). Some classes of regular univalent functions, A Proceedings of the American Mathematical Society, 16, 755–758.

[12] Ruscheweyh, S. (1975). New criteria for univalent functions, Proceedings of the American Mathematical Society, vol. 49, 109–115.

[13] Srivastava, H. M., Qureshi, M. I., Malik, S. H. (2021). Some hypergeometric transformations and reduction formulas for the Gauss function and their applications involving the Clausen function, J. Nonlinear Var. Anal.,5, 981–987.

[14] Abu Risha, M. H., Annaby, M. H., Ismail, Z. S., Mansour, Z. S. (2007). Linear q–difference equations, Zeitschrift fur Analysis und ihre Anwendungen, 26, 481–494.

[15] Gasper, G., Rahman, M. (1990). Basic Hypergeometric Series, Cambridge University Press, Cambridge.

[16] Jackson, F. H. (1905). The application of basic numbers to Bessel's and Legendre's functions, Proceedings of the London Mathematical Society, 3, 1–23.

[17] Ekram E. Ali, Rabha M. El-Ashwah, Abeer M. Albalahi. (2025). Geometric applications for meromorphic functions involving -linear operators. *AIMS Mathematics*, 10(12): 30990-31009.

[18] MacGregor, T. H. (1967). Majorization by univalent functions, Duke Math. J., 34, 95–102.

[19] Tang, H., Srivastava, H. M., Li, S. H., Deng, G. T. (2020). Majorization results for subclasses of starlike functions based on the sine and cosine functions, Bull. Iran. Math. Soc., 46, 381–388. 6 Proceedings of International Conference on Mathematical Advances and Applications.

[20] Arif, M., Ul-Haq, M., Barukab, O., Khan, Abullah, S. A. S. (2021). Majorization results for certain subfamilies of analytic functions, J. Funct. Spaces, 2021, 1–6.

[21] Cho, N. E., Oroujy, Z. E., Adegani, A., Ebadian, A. (2020). Majorization and coefcient problems for a general class of starlike functions, Symmetry, 12, 476.

[22] Tang, H., Deng, G. (2018). Majorization problems for two subclasses of analytic functions connected with the Liu–Owa integral operator and exponential function, J. Inequalities Appl., 2018, 277–311.

[23] Altıntaş, O., Srivastava, H. M. (2001). Some majorization problems associated with p-valently starlike and convex functions of complex order, East Asian Math. J., 17, 175–183.

[24] Altıntas¸, O., Ozkan, O., Srivastava, H. M. (2001). Majorization by starlike functions of complex order, Complex Variables, Theory and Appl.: An Int. J., 46, 207–218.

[25] Dhuria, K., Mathur, R. (2013). Majorization for certain classes of meromorphic functions defined by integral operator, International Journal of Open Problems in Complex Analysis, 5, 50–56.

[26] Goyal, S. P., Goswami, P. (2012). Majorization for certain classes of meromorphic functions defined by integral operator, Annales UMCS, Math., 66, 57–62.

[27] Bulut, S., Adegani, E. A., Bulboaca, T. (2021). Majorization results for a general subclass of meromorphic multivalent functions, Oliteh. Univ. Buchar. Sci. Bull. Ser. A Appl. Math. Phys., 83, 121–128.

[28] Janani, T., Murugusundaramoorthy, G. (2014). Majorization problems for p-valently meromorphic functions of complex order involving certain integral operator, Glob. J. Math. Anal., 2, 146–151.

[29] Tang, H. Aouf, M. K., Deng, G. (2015). Majorization problems for certain subclasses of meromorphic multivalent functions associated with the Liu-Srivastava operator, Filomat, 29, 763–772.

[30] Rasheed, A., Hussain, S., Ghoos S., Shah, A., Darus, M., Lodhi, S. (2020). Majorization problem for two subclasses of meromorphic functions associated with a convolution operator, AIMS Mathematics, 5, 5157–5170.

[31] Panigrahi, T., El-Ashwah, R. (2017). Majorization for subclasses of multivalent meromorphic functions defined through iterations and combinations of the Liu-Srivastava operator and a meromorphic analogue of the Cho-Kwon-Srivastava operator, Filomat, 31, 6357–6365.

[32] Deniz, E., Kazımoglu, S., Kızıltepe, A. (2023). Majorization Properties for a Subclass of Analytic Function of Complex Order, Conference Proceeding Science and Technology, 1, 254–265.

[33] Janowski, W. (1973). Some extremal problems for certain families of analytic functions, Ann. Pol. Math., 28, 297–326.

[34] Nehari, Z. (1955). Conformal Mappings, MacGrow-Hill Book Company, New york, Torinto and London.

[35] Selvakumaran, K. A., Purohit, S. D., Secer, A. (2014). Majorization for a class of analytic functions defined by q-diferentiation, Math. Probl. Eng., Article ID 653917, 5 pages.

[36] Vijaya, K., Murugusundaramoorthy, G., Cho, N. E. (2021). Majorization Problems for Uniformly Starlike functions based on Ruscheweyh q-diferential operator defined with exponential function, Nonlinear Funct. Anal. Appl., 26, 71–81.