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SOME MATHEMATICAL NOTATIONS FOR THE COLLATZ PROCEDURE

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ABSTRACT

The Collatz Conjecture is more than a theoretical problem in mathematics; it also holds significant value in practical computational contexts. Beyond its abstract mathematical nature, the conjecture serves as a versatile tool within Computer Engineering and Computer Science. The algorithm derived from the Collatz process has been applied in several emerging fields such as steganography, cryptology, data hiding, and digital watermarking, demonstrating its adaptability to real-world problems. This study investigates these application areas in detail and introduces a number of mathematical expressions formulated through the Collatz Procedure. These expressions have the potential to support both theoretical efforts aimed at proving the conjecture and practical implementtations across various domains. By bridging pure mathematics with applied computer science, the Collatz Conjecture continues to inspire interdisciplinary research and innovation.

1. Introduction

The Collatz Conjecture was first proposed by the German mathematician Lothar Collatz in 1937 and became very popular in the mathematical community. Although this conjecture is based on a very simple rule, it is still considered to be one of the most important unsolved problems since no definitive proof has been found to date. The conjecture asserts that for any positive integer n the repeated application of a defined function will always lead to the number 1 [2, 4, 9].

In this context, the Collatz function is defined as follows: If n is an odd number $3n + 1$, if it is an even number $n/2$. These operations are continued by repeating the same rules on the new number obtained. The claim is that no matter which positive integer is initially chosen, 1 will always be reached at the end of this iterative process. This property generates sequences of numbers called "Collatz trajectories".

Theoretical studies on the proof of the conjecture have been carried out by many mathematicians using different methods. Some of these studies involve number theory, while others involve computational methods and data visualization techniques. For example, in [6], theoretical approaches are presented in which the conjecture is analyzed under certain classes, while in [7], a representation of the problem in the 4 (quadratic) number system is given. Thanks to this representation, the behavior of the numbers can be analyzed in a simpler way. Then, in [7], based on this representation, the problem is modeled in the context of graph theory and the graphical relationships of the trajectories are presented.

The difficulty of the conjecture stems not only from the technical complexity but also from the resistance to mathematical proof of the simplicity of its structure. This is summarized in the words of the famous Hungarian mathematician Paul Erdős: “Mathematics is not yet ready for this type of problem.” [2]. This statement implies that the conjecture needs to be addressed not only with existing mathematical tools, but perhaps with new methods not yet developed.

One of the most remarkable studies on the Collatz Conjecture in recent years has been carried out by Fields Medalist Terence Tao. In his 2019 paper, Tao showed that the vast majority of Collatz orbits behave under a certain probabilistic constraint [10]. Although this work does not provide an absolute proof, it provides strong evidence for the statistical correctness of the conjecture and provides a new perspective to analyze the problem.

The Collatz Conjecture is not only limited to mathematical theories, but is also of interest in different disciplines such as computer science, cryptography and steganography. In particular, the use of Collatz-based algorithms has been proposed in areas such as data security, digital authentication and information hiding (steganography). In such applications, the deterministic but complex structure of Collatz sequences provides advantages for both encryption and authentication processes. For example, in encryption algorithms, Collatz-based operation produces outputs that are more difficult to predict than classical linear algorithms, thereby increasing the level of security.

In conclusion, the Collatz Conjecture is both a problem that still remains to be solved in the depths of mathematical theory and a source of inspiration for interdisciplinary studies with its potential to offer practical benefits in current technological applications. Current research not only focuses on the proof of the conjecture, but also explores how this theoretical construct can be applied in different fields. This shows that the Collatz Conjecture has become more than just a mathematical curiosity and has become a multifaceted object of scientific research.

2. Application Areas of the Collatz Conjecture: An Interdisciplinary Approach

The Collatz Conjecture attracts attention not only for its mathematical foundations but also for its applications in various industrial and technological fields. Especially in computer engineering, cryptography and data security, practical solutions are developed by utilizing the structural properties of Collatz sequences. In this context, the deterministic but chaotic structure of the Collatz Conjecture provides both theoretical and practical contributions in various digital security applications.

2.1 Steganography and Digital Watermarking

The Collatz Conjecture has been effectively used in steganography and content protection (digital watermarking) applications on digital media. Applied to multimedia data such as images, audio and video, these methods serve various security purposes such as copyright protection, data integrity verification and identification.

In particular, the iterative nature of Collatz sequences allows data to be stored in a certain order but in a way that is difficult to understand from the outside. In this context, the trajectories generated by the algorithm can be integrated into data storage processes to increase security and improve the performance of existing steganographic techniques [5].

2.2 Cryptography and Encryption Technologies

The Collatz Conjecture is also considered as a potential component in modern encryption algorithms due to its complex dynamic structure. Especially in symmetric and asymmetric key generation, the chaotic behavior of Collatz sequences is used to reduce the predictability of encryption keys.

In specialized fields such as image cryptography, Collatz-based operations contribute to making data encryption more secure. Thanks to these methods, both a high level of security is achieved and the robustness of encryption processes against artificial intelligence is increased [1, 3, 8, 11].

2.3 Pseudo-Random Number Generation (PRNG)

The irregular and unpredictable structure of the Collatz sequence provides a suitable basis for pseudo-random number generators (PRNGs). Although the sequences are not truly random, their complexity and irregularity can be used to generate near-random sequences.

This property offers significant advantages in high-performance PRNG designs, especially in simulations, statistical sampling and cryptosystems. Studies have shown that Collatz-based PRNG algorithms are remarkable in terms of both efficiency and security [3].

2.4 Graph Theory and Network Analysis

Collatz trajectories can be modeled as directed graphs. In these structures, each number is represented as a node and the transformations between them as edges. Such graphical representations are particularly useful for modeling computational complexity, network behavior and optimization problems.

Collatz graphs are considered to be a powerful tool in algorithm design and analysis, as they clearly show the transition relations between numbers. Moreover, the fractal-like properties of these graphical structures have been the subject of several researches for both visual analysis and structural classifications [12].

3. Collatz Conjecture

3.1 Definition of the Problem

The Collatz Conjecture is a mathematical conjecture that predicts that successive applications to a given function starting from any positive integer will always lead to 1. This hypothesis, first proposed by Lothar Collatz in 1937, has not been confirmed by a general proof despite extensive numerical verifications.

By definition, a given positive integer n is subject to the function defined as follows:

$f: Z^+ \to Z^+$

$f_0 = n \neq 0$

$$f_{i+1} = \begin{cases} \frac{f_i}{2}, & \text{if } f_i \text{ is an even number} \\ 3f_i + 1, & \text{if } f_i \text{ is an odd number} \end{cases}$$

This definition continues the process by re-entering the number into the function each time and the process continues until $f_i = 1$. The sequence of numbers generated in this iterative process is called the "Collatz trajectory".

The basic assumption of the problem is the following:

No matter which positive integer is chosen initially, the above operations will always reach 1 after a certain number of steps.

3.2 Collatz Function

In order to express the Collatz Conjecture more systematically, the functional definition can be given as follows:

$C: Z^+ \to Z^+$

$$C(n) = \begin{cases} \frac{n}{2}, & n \to n \equiv 0(mod\ 2) \\ 3n+1 & n \to n \equiv 1(mod\ 2) \end{cases}$$

This function generates a sequence by successively applying it to itself:

$$F(n) = \left\{n, C(n), C\big(C(n)\big), C\left(C\big(C(n)\big)\right), \ldots\right\} = \{n, C(n), C^2(n), C^3(n), \ldots\}$$

At each step a new term is obtained by reapplying the function to the previous result. The assumption is that this sequence must necessarily sum to 1.

3.3 Collatz Procedure

The implementation of the Collatz function is as follows step by step:

1. Initially a positive integer is chosen.
2. If the number is even, it is divided by two.
3. If the number is odd, it is first multiplied by three and then one is added.
4. This is repeated until the number is 1.

Example:

Taking *n* = 7 as the initial value, the process steps are as follows:

1. 7 (odd) → 3(7) + 1 = 22

2. 22 (even) → 22 / 2 = 11

3. 11 (odd) → 3(11) + 1 = 34

4. 34 (even) → 34 / 2 = 17

5. 17 (odd) → 3(17) + 1 = 52

6. 52 (even) → 52 / 2 = 26

7. 26 (even) → 26 / 2 = 13

8. 13 (odd) → 3(13) + 1 = 40

9. 40 (even) → 40 / 2 = 20

10. 20 (even) → 20 / 2 = 10

11. 10 (even) → 10 / 2 = 5

12. 5 (odd) → 3(5) + 1 = 16

13. 16 (even) → 16 / 2 = 8

14. 8 (even) → 8 / 2 = 4

15. 4 (even) → 4 / 2 = 2

16. 2 (even) → 2 / 2 = 1

Result: 7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

Collatz Orbits of Numbers from 1 to 10:

C(1) = { 1 },

C(2) = { 2, 1 } ,

C(3) = { 3, 10, 5, 16, 8, 4, 2, 1},

C(4) = { 4, 2, 1},

C(5) = { 5, 16, 8, 4, 2, 1},

C(6) = { 6, 3, 10, 5, 16, 8, 4, 2, 1},

C(7) = { 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1},

C(8) = { 8, 4, 2, 1},

C(9) = { 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1},

C(10) = { 10, 5, 16, 8, 4, 2, 1 }

3.4 Notational Representation of the Conjecture

The Collatz Conjecture for the set of all positive integers is expressed as follows:

$$\forall n \in Z^+, k \in Z^+ \text{so that, } \exists C^k(n) = 1$$

This expression states that for every positive integer n, there will be a number of steps k such that $C^k(n) = 1$. This is the mathematical essence of the conjecture.

3.5 Numerical Verification and Computerized Control

As of 2020, the Collatz Conjecture has been tested using computers up to very large numbers. The Collatz trajectories of all positive integers up to approximately 2^{68} ($2^{68} \approx 2.95 \times 10^{20}$) have been calculated and the result has reached 1 every time. These extensive calculations strongly support the accuracy of the conjecture.

However, these tests are limited in scope and pertain only to a certain range. Therefore, unless it is supported by a generally valid mathematical proof, the Collatz Conjecture still has to be considered as "a mathematical hypothesis whose truth is supported by strong evidence but has not yet been proven".

4. Some Mathematical Representations for the Collatz Conjecture

In the following sections of our paper, we will disregard even numbers; all operations will be presented using only odd numbers. For example, if we consider the Collatz Procedure for the number $n = 9$, it generally takes 19 steps to reach the number 1:

C (9) = {9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1}

However, if we represent this procedure using only odd numbers, we would reach the number 1 in just 6 steps:

CO (9) = {9, 7, 11, 17, 13, 5, 1}

Here, the subset of the set C consisting of only odd numbers is denoted by CO.

In this case, the following theorem holds:

Theorem 1: For every number n, the following expression holds:

$$3^m \cdot \boldsymbol{n} + 3^{m-1} + 3^{m-2} \cdot 2^{k_{m-2}} + 3^{m-3} \cdot 2^{k_{m-3}} + \cdots + 3^2 \cdot 2^{k_2} + 3^1 \cdot 2^{k_1} + 3^0 \cdot 2^{k_0} = 2^k \quad (1)$$

Here, m is the number of steps in CO(n), and $k_0 = k - 4$, $k_0 > k_1 > k_2 > \cdots > k_{m-2}$. Below is the expansion of expression (1) for all odd numbers in the range $n = 1$ to 27:

$3^1 \cdot \mathbf{1} + 3^0 = 3 + 1 = 4 = 2^2$

$3^2 \cdot \mathbf{3} + 3^1 + 3^0 \cdot 2^1 = 9 \cdot 3 + 3 + 1 \cdot 2 = 27 + 3 + 2 = 32 = 2^5$

$3^1 \cdot \mathbf{5} + 3^0 = 15 + 1 = 16 = 2^4$

$$3^5 \cdot \mathbf{7} + 3^4 + 3^3 \cdot 2 + 3^2 \cdot 2^2 + 3^1 \cdot 2^4 + 2^7 = 243 \cdot 7 + 81 + 27 \cdot 2 + 9 \cdot 24 + 3 \cdot 16 + 128$$
$$= 1702 + 81 + 54 + 36 + 48 + 128 = 2048 = 2^{11}$$

$3^6 \cdot \mathbf{9} + 3^5 + 3^4 \cdot 2^2 + 3^3 \cdot 2^3 + 3^2 \cdot 2^4 + 3 \cdot 2^6 + 2^9 = 2^{13}$

$3^4 \cdot \mathbf{11} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^3 + 2^6 = 2^{10}$

$3^2 \cdot \mathbf{13} + 3 + 2^3 = 2^7$

$3^5 \cdot \mathbf{15} + 3^4 + 3^3 \cdot 2 + 3^2 \cdot 2^2 + 3 \cdot 2^3 + 2^8 = 2^{12}$

$3^3 \cdot \mathbf{17} + 3^2 + 3 \cdot 2^2 + 2^5 = 2^9$

$3^6 \cdot \mathbf{19} + 3^5 + 3^4 \cdot 2^1 + 3^3 \cdot 2^4 + 3^3 \cdot 2^5 + 3 \cdot 2^7 + 2^{10} = 2^{14}$

$3^1 \cdot \mathbf{21} + 3^0 = 63 + 1 = 64 = 2^6$

$3^4 \cdot \mathbf{23} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^2 + 2^7 = 2^{11}$

$3^7 \cdot \mathbf{25} + 3^6 + 3^5 \cdot 2^2 + 3^4 \cdot 2^3 + 3^3 \cdot 2^6 + 3^2 \cdot 2^7 + 3 \cdot 2^9 + 2^{12} = 2^{16}$

$$3^{41} \cdot \mathbf{27} + 3^{40} + 3^{39} \cdot 2 + 3^{38} \cdot 2^3 + 3^{37} \cdot 2^4 + 3^{36} \cdot 2^5 + 3^{35} \cdot 2^6 + 3^{34} \cdot 2^7 + 3^{33} \cdot 2^9 + 3^{32} \cdot 2^{11}$$
$$+ 3^{31} \cdot 2^{12} + 3^{30} \cdot 2^{14} + 3^{29} \cdot 2^{15} + 3^{28} \cdot 2^{16} + 3^{27} \cdot 2^{18} + 3^{26} \cdot 2^{19} + 3^{25} \cdot 2^{20}$$
$$+ 3^{24} \cdot 2^{21} + 3^{23} \cdot 2^{23} + 3^{22} \cdot 2^{26} + 3^{21} \cdot 2^{27} + 3^{20} \cdot 2^{28} + 3^{19} \cdot 2^{30} + 3^{18} \cdot 2^{31}$$
$$+ 3^{17} \cdot 2^{33} + 3^{16} \cdot 2^{34} + 3^{15} \cdot 2^{35} + 3^{14} \cdot 2^{36} + 3^{13} \cdot 2^{37} + 3^{12} \cdot 2^{38} + 3^{11} \cdot 2^{41}$$
$$+ 3^{10} \cdot 2^{42} + 3^9 \cdot 2^{43} + 3^8 \cdot 2^{44} + 3^7 \cdot 2^{48} + 3^6 \cdot 2^{50} + 3^5 \cdot 2^{52} + 3^4 \cdot 2^{56} + 3^3$$
$$\cdot 2^{59} + 3^2 \cdot 2^{60} + 3 \cdot 2^{61} + 2^{66} = 2^{70}$$

We can also write formula (1) in the following way:

$$2\text{^}k - 3\text{^}m \cdot \boldsymbol{n} - 3\text{^}(m-1) - 2\text{^}(k-4) = 3\text{^}(m-2) \cdot 2\text{^}(k\_(m-2)\ ) + 3\text{^}(m-3) \cdot 2\text{^}(k\_(m-3)\ ) +$$
$$\cdots + 3\text{^}2 \cdot 2\text{^}(k\_2\ ) + 3\text{^}1 \cdot 2\text{^}(k\_1\ )\ (2)$$

Below, by analyzing the above formulas, we will show how they are derived:

First, let's consider the odd numbers up to 27 that reach the number 1 in just 1 step within the set CO (that is, where $m = 1$); these numbers are $n = 1$, 5, and 21:

$$n = 1 \rightarrow 3 \cdot \mathbf{1} + 1 = 4 \rightarrow 3^1 \cdot \mathbf{1} + 3^0 = 2^2$$

$$n = \mathbf{5} \rightarrow 3 \cdot \mathbf{5} + 1 = 15 + 1 = 16 \rightarrow 3^1 \cdot \mathbf{5} + 3^0 = 2^4$$

$$n = \mathbf{21} \rightarrow 3 \cdot \mathbf{21} + 1 = 63 + 1 = 64 \rightarrow 3^1 \cdot \mathbf{21} + 3^0 = 2^6$$

Now, let's consider the odd numbers up to 27 that reach the number 1 in 2 steps within the set CO (that is, where $m = 2$); these numbers are $n = 3$ and 13:

$$n = \mathbf{3} \rightarrow 3 \cdot \mathbf{3} + 1 = 10 \rightarrow 10/2 = 5 \rightarrow 3 \cdot 5 + 1 = 16 \rightarrow 3 \cdot ((3 \cdot \mathbf{3} + 1)/2) + 1 = 16 \rightarrow 3^2 \cdot \mathbf{3} + 3 \cdot 1 + 2 = 16 \cdot 2 = 32 \rightarrow 3^2 \cdot \mathbf{3} + 3^1 + 3^0 \cdot 2^1 = 2^5$$

$$n = \mathbf{13} \rightarrow 3 \cdot \mathbf{13} + 1 = 40 \rightarrow 40/8 = 5 \rightarrow 3 \cdot 5 + 1 = 16 \rightarrow 3 \cdot ((3 \cdot \mathbf{13} + 1)/8) + 1 = 16 \rightarrow 3^2 \cdot \mathbf{13} + 3 \cdot 1 + 8 = 16 \cdot 8 = 128 \rightarrow 3^2 \cdot \mathbf{13} + 3^1 + 3^0 \cdot 2^3 = 2^7$$

Only the number 17 among the odd numbers up to 27 reaches the number 1 in 3 steps within the set CO (that is, $m = 3$). Let's consider the number $n = 17$:

$$n = 17 \rightarrow 3 \cdot \mathbf{17} + 1 = 52 \rightarrow 52/4 = 13 \rightarrow 3 \cdot 13 + 1 = 40 \rightarrow 40/8 = 5 \rightarrow$$

$$3 \cdot 5 + 1 = 16 \rightarrow 3 \cdot ((3 \cdot 13 + 1)/8) + 1 = 16 \rightarrow 3^2 \cdot 13 + 3 \cdot 1 + 8 = 16 \cdot 8 = 128$$

$$\rightarrow 3^2 \cdot 13 + 3^1 + 3^0 \cdot 2^3 = 2^7 \rightarrow 3^2(3 \cdot \mathbf{17} + 1)/4 + 3^1 + 3^0 \cdot 2^3 = 2^7$$

$$\rightarrow 3^2(3 \cdot \mathbf{17} + 1)/4 + 3^1 + 3^0 \cdot 2^3 = 2^7 \rightarrow 3^3 \cdot \mathbf{17} + 3^2 + 3 \cdot 2^2 + 3^0 \cdot 2^5 = 2^9$$

Now, let's consider the odd numbers up to 27 that reach the number 1 in 4 steps within the set CO (that is, $m = 4$); these numbers are $n = 11$ and 23:

$$n = \mathbf{11} \rightarrow 3 \cdot \mathbf{11} + 1 = 34 \rightarrow 34/2 = 17 \rightarrow 3 \cdot 17 + 1 = 52 \rightarrow 52/4 = 13$$

$$\rightarrow 3 \cdot 13 + 140 \rightarrow 40/8 = 5 \rightarrow 3 \cdot 5 + 1 = 16 \rightarrow 3 \cdot ((3 \cdot 13 + 1)/8) + 1 = 16$$

$$\rightarrow 3^2 \cdot 13 + 3 \cdot 1 + 8 = 16 \cdot 8 = 128 \rightarrow 3^2 \cdot 13 + 3^1 + 3^0 \cdot 2^3 = 2^7$$

$$\rightarrow 3^2(3 \cdot 17 + 1)/4 + 3^1 + 3^0 \cdot 2^3 = 2^7 \rightarrow 3^3 \cdot 17 + 3^2 + 3 \cdot 2^2 + 3^0 \cdot 2^5 = 2^9$$

$$\rightarrow 3^3 \cdot (3 \cdot \mathbf{11} + 1)/2 + 3^2 + 3 \cdot 2^2 + 3^0 \cdot 2^5 = 2^9$$

$$\rightarrow 3^4 \cdot \mathbf{11} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^3 + 3^0 \cdot 2^6 = 2^{10}$$

$$\rightarrow 3^4 \cdot \mathbf{11} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^3 + 2^6 = 2^{10}$$

$$n = \mathbf{23} \rightarrow 3 \cdot \mathbf{23} + 1 = 70 \rightarrow 70/2 = 35 \rightarrow 3 \cdot 35 + 1 = 106 \rightarrow 106/2 = 53$$

$$\rightarrow 3 \cdot 53 + 1 + 32 = 16 \cdot 32 = 512 \rightarrow 3^2 \cdot 53 + 3^1 + 3^0 \cdot 2^5 = 2^9$$

$$\rightarrow 3^2(3 \cdot 35 + 1)/2 + 3^1 + 3^0 \cdot 2^5 = 2^9 \rightarrow 3^3 \cdot 35 + 3^2 + 3 \cdot 2^1 + 3^0 \cdot 2^6 = 2^{10}$$

$$\rightarrow 3^3 \cdot (3 \cdot \mathbf{23} + 1)/2 + 3^2 + 3 \cdot 2^1 + 3^0 \cdot 2^6 = 2^{10}$$

$$\rightarrow 3^4 \cdot \mathbf{23} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^2 + 3^0 \cdot 2^7 = 2^{11}$$

$$\rightarrow 3^4 \cdot \mathbf{23} + 3^3 + 3^2 \cdot 2 + 3 \cdot 2^2 + 2^7 = 2^{11}$$

The remaining numbers can also be analyzed similarly by examining the formulas above to show how they are derived.

Theorem 2: For every number *m*, the following identity holds:

$$3^m + 3^{m-1} \cdot 2^0 + 3^{m-2} \cdot 2^2 + 3^{m-3} \cdot 2^4 + 3^{m-4} \cdot 2^6 + \cdots + 3^1 \cdot 2^{2m-4} + 3^0 \cdot 2^{2m-2} = 2^{2m} \quad (3)$$

Below is the expansion of expression (3) for the values $m = 1, 2, 3$:

$3^1 + 3^0 \cdot 2^0 = 3 + 1 \cdot 1 = 3 + 1 = 4 = 2^2 = 2^{2\cdot 1}$

$3^2 + 3^1 \cdot 2^0 + 3^0 \cdot 2^2 = 9 + 3 \cdot 1 + 1 \cdot 4 = 9 + 3 + 4 = 16 = 2^4 = 2^{2\cdot 2}$

$3^3 + 3^2 \cdot 2^0 + 3^1 \cdot 2^2 + 3^0 \cdot 2^4 = 27 + 9 \cdot 1 + 3 \cdot 4 + 1 \cdot 16 =$

$= 27 + 9 + 12 + 16 = 64 = 2^6 = 2^{2\cdot 3}$

By using formula (1), we can write the following expression for the number.

Theorem 3:

$$n = \frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{3} \dots \frac{1}{3}\left(2^k - 2^{k_0}\right) - 2^{k_1}\right) - 2^{k_2}\right) - \cdots - 2^{k_{m-3}}\Big) - 2^{k_{m-2}}\Big) - 1) \quad (4)$$

Here, *m* is the number of distinct elements in CO(*n*) excluding *n* itself, and the number of parentheses and the terms "1/3" are also equal to *m*.

Below is the expansion of expression (4) for $n = 9$:

$3^6 \cdot \mathbf{9} + 3^5 + 3^4 \cdot 2^2 + 3^3 \cdot 2^3 + 3^2 \cdot 2^4 + 3 \cdot 2^6 + 2^9 = 2^{13} \Rightarrow$

$3^5 \cdot \mathbf{9} + 3^4 + 3^3 \cdot 2^2 + 3^2 \cdot 2^3 + 3^1 \cdot 2^4 + 2^6 = \frac{1}{3}(2^{13} - 2^9) \Rightarrow$

$3^4 \cdot \mathbf{9} + 3^3 + 3^2 \cdot 2^2 + 3^1 \cdot 2^3 + 2^4 = \frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) \Rightarrow$

$3^3 \cdot \mathbf{9} + 3^2 + 3^1 \cdot 2^2 + 2^3 = \frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) - 2^4\right) \Rightarrow$

$3^2 \cdot \mathbf{9} + 3^1 + 2^2 = \frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) - 2^4) - 2^3) \Rightarrow$

$3 \cdot \mathbf{9} + 1 = \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) - 2^4) - 2^3) - 2^2) \Rightarrow$

$3 \cdot \mathbf{9} = \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) - 2^4) - 2^3) - 2^2) - 1 \Rightarrow$

$\mathbf{9} = \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3}(2^{13} - 2^9) - 2^6\right) - 2^4) - 2^3) - 2^2) - 1) \Rightarrow$

$= \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3}(8192 - 512) - 64\right) - 16) - 8) - 4) - 1) \Rightarrow$

$= \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}\left(\frac{1}{3} \cdot 7680 - 64\right) - 16) - 8) - 4) - 1) \Rightarrow$

$= \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3} \cdot 2496 - 16) - 8) - 4) - 1) =$

$= \frac{1}{3}(\frac{1}{3}(\frac{1}{3}(\frac{1}{3} \cdot 816 - 8) - 4) - 1) =$

$= \frac{1}{3}(\frac{1}{3}(\frac{1}{3} \cdot 264 - 4) - 1) =$

$= \frac{1}{3}(\frac{1}{3} \cdot 84 - 1) =$

$= \frac{1}{3} \cdot 27$

Since there are 6 elements in CO(9) different from 9, the above expansion for the number 9 contains 6 instances of "1/3" and 6 parentheses.

6. Conclusion

Although the Collatz Conjecture has not yet been conclusively proven from a mathematical standpoint, its structural properties have enabled significant applications across various fields, particularly in computer science, algorithm design, cryptography, and data security. This study has explored the fundamental mathematical basis of the conjecture and highlighted its potential uses in areas such as steganography, digital watermarking, chaotic encryption systems, and pseudo-random number generation.

The findings suggest that the Collatz Conjecture is more than just a theoretical curiosity it represents a robust mathematical construct with practical implications in modern computational technologies. Due to its theoretical depth and practical relevance, the Collatz Conjecture continues to hold a notable position in both mathematics and computer science.

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